Delayed Retirement or More Births? Short-Run Relief and Long-Run Sustainability of China's Pension System

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Abstract

We build a heterogeneous-agent overlapping generations model of China's combined-accounts pension system to assess its short-run and long-run fiscal sustainability under population aging. Under current policy, the pension deficit widens to 6.4% of GDP in the long run. Among feasible reforms, delayed retirement provides immediate fiscal relief but limited long-run help, whereas pro-natalist policy improves long-run sustainability only after a two-decade lag. The two policies are complementary. We introduce the fiscal value of birth as the present-value net fiscal gain from an additional birth, providing a break-even benchmark for pro-natalist subsidies. At TFR of 1.3–1.5, each additional birth is worth 139,000–149,000 yuan.

Keywords: Heterogeneous Agent, Social Security, Sustainability, Population Aging, OLG Model

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1 Introduction

China is experiencing a rapid demographic transition marked by very low fertility and accelerating population aging. The total fertility rate has fallen to approximately 1.0, less than half the replacement level, while the population aged 65 and above reached 220 million—15.6% of the total—in 2024. As the 1962–1973 baby-boom cohorts enter retirement, the old-age dependency ratio is projected to rise sharply, putting sustained pressure on the basic pension system. The system covers roughly one billion people, already runs an annual cash-flow deficit of 1.3 trillion yuan, and, under baseline projections, faces exhaustion of its accumulated surplus by the mid-2040s. Policy responses have already moved from debate to implementation: beginning in 2025, China started a gradual delayed-retirement reform, and a national child-rearing subsidy for children under age three applies from January 1, 2025.

Although these reforms are now in place, their fiscal effects and optimal design remain quantitative questions. The relevant margins operate at different horizons: delayed retirement affects contribution revenue and benefit duration immediately, benefit adjustment propagates through the existing stock of pension claims, and higher fertility expands the contributor base only after new cohorts enter the labor force. In China, these margins also interact with a combined-accounts benefit formula tied to individual account balances and wage histories. A credible sustainability assessment therefore needs to go beyond aggregate dependency ratios or uniform-transfer pension approximations and follow households, pension finances, and fiscal closure along the demographic transition path. This paper quantifies the fiscal adjustment required to keep China's pension system sustainable and evaluates how delayed retirement, benefit adjustment, and higher fertility improve sustainability over the short and long run.

We make three contributions. *First*, we develop, to our knowledge, the first heterogeneous-agent general-equilibrium analysis of China's combined-accounts pension system with transition dynamics. The model tracks personal-account balances and historical wages at the individual level, allowing it to implement China's statutory combined-accounts benefit formula rather than approximate pensions as uniform transfers. *Second*, we rank three reform instruments—reduced pooling, delayed retirement, and higher fertility—by their effects on the wage tax in both the short run and the long run. Short-run effects are governed by fiscal inertia: the pension benefits of currently living cohorts are pre-determined by their accumulated state variables, so benefit-formula changes propagate only gradually through the existing distribution of pension claims. Delayed retirement and higher fertility are complementary because they act on disjoint margins of the pension system. *Third*, we introduce the *fiscal value of birth* as a policy metric for evaluating pro-natalist policy in general equilibrium. Rather than estimating an endogenous fertility response, we treat fertility as a policy target and compute the present-value net fiscal gain per additional birth, providing a break-even benchmark for child-rearing subsidies.

The model combines the lifecycle structure of Auerbach and Kotlikoff [1987] with the incomplete-market household heterogeneity of Aiyagari [1994]. It represents China's segmented pension system by distinguishing the urban employee combined-accounts scheme from the urban–rural resident pension scheme, and by allowing workers to differ by pension participation and urban–rural status. The urban benefit formula combines an individual-account annuity with a pooling component indexed to wage history, so the model carries the pension-state variables needed to capture fiscal inertia during the transition. We solve the general-equilibrium transition path from a 2023 initial condition over 200 years of demographic change, with the wage tax adjusting endogenously to maintain the debt-to-GDP target. The policy experiments compare delayed retirement, reduced pooling transfers, and higher fertility, both separately and in combination.

Three quantitative findings stand out. First, under current policy the pension deficit widens to 6.4% of GDP in the long run, requiring the wage tax to rise from 9.6% to 27.5%. Second, the three reform instruments form a clear ranking. Reduced pooling reduces the transitional wage tax by 2.8 pp but cannot restore long-run sustainability (terminal tax rate 23.6%) and hurts the low earners who rely most on the pooling component. Delayed retirement to age 65 extends the contribution horizon and shortens the retirement phase, reducing the transitional wage tax by 3.3 pp and the terminal rate by 5.1 pp. Higher fertility expands the future contribution base only after a two-decade lag; in the long run it yields the largest single-instrument dividend (−9.0 pp terminal). Combining delayed retirement to age 65 with replacement-level fertility cuts the terminal wage tax by 11.7 pp—the two instruments are complementary because they act on disjoint margins of the pension system. Third, the marginal fiscal value of birth is *highest* at low fertility levels near the current TFR of 1.0, where the first additional births alleviate the most acute dependency pressure. In the policy-relevant range of TFR 1.3–1.5, this marginal value justifies subsidies of approximately 139,000–149,000 yuan per additional birth, or equivalently annual childcare transfers of approximately 48,000–52,000 yuan over the first three years of life.

1.1 Related Literature

The paper sits at the intersection of three literatures.

Heterogeneous-agent OLG fiscal sustainability. The paper first builds on the quantitative OLG tradition for fiscal sustainability and social-security reform. The framework initiated by Auerbach and Kotlikoff [1987] was enriched by incomplete-market household heterogeneity [Bewley, 1986, Aiyagari, 1994, Huggett, 1996]; Krueger et al. [2016] survey the modern heterogeneous-agent macroeconomic approach. Applied work on Social Security reform computes explicit transition paths and policy tradeoffs, including Huang et al. [1997], De Nardi et al. [1999], and Kitao [2014] for the United States, and İmrohoroğlu and Kitao [2012], İmrohoroğlu et al. [2016] for Japan. Related contributions study the welfare and distributional consequences of social-security design under incomplete markets [Conesa and Krueger, 1999, Krueger and Kubler, 2006, Nishiyama and Smetters, 2007, Krueger and Ludwig, 2007, Ludwig et al., 2009, Fehr et al., 2013]. Our paper extends this transition-computation tradition to China's institutionally distinctive combined-accounts pension system and tracks the individual-level state variables (b, e) required by the statutory benefit formula.

China's pension system and demographic transition. The paper also relates to quantitative studies of China's pension system under rapid demographic change. Fang and Feng [2018] provide an institutional overview of China's fragmented multi-pillar pension architecture. Song et al. [2015] study intergenerational sharing of growth under demographic transition in an OLG framework calibrated to China, and He et al. [2019] analyze how rapid aging and pension reform affect saving and labor supply. İmrohoroğlu and Zhao [2018, 2020] embed family insurance and financial constraints in lifecycle models of Chinese saving, while Bairoliya et al. [2018] evaluate rural health-insurance and pension reforms and Gai et al. [2025] study the rural pension scheme's effects on labor reallocation and aggregate income. Relative to this literature, our model focuses on the fiscal sustainability of the combined-accounts pension system and follows the transition path from the current demographic and fiscal environment.

Pension reform instruments in China. Finally, the paper speaks to work evaluating specific reform instruments for China's pension system. Deng et al. [2023] compare delayed retirement with benefit adjustment in a life-cycle model with skill groups, highlighting the labor-supply and welfare tradeoffs of alternative reforms. A broader Chinese policy literature examines contribution-rate design, replacement-rate adjustment, delayed retirement, and fiscal sustainability [Li, 2024, Lü et al., 2024]. Our contribution is to compare delayed retirement, reduced pooling transfers, and higher fertility within a single heterogeneous-agent general-equilibrium framework, and to show how their effectiveness differs between short-run fiscal pressure, long-run sustainability, and intergenerational incidence.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 describes the calibration of structural parameters, the matching of 2023 targeted moments, and the out-of-sample validation against the 2024 deficit. Section 4 reports quantitative results on policy experiments, distributional effects, and welfare. Section 5 quantifies the fiscal value of each additional birth. Section 6 concludes.

2 Theoretical Model

This section presents the quantitative environment used to evaluate China's pension sustainability and reform experiments. The model is a heterogeneous-agent overlapping-generations economy with incomplete markets, idiosyncratic labor-productivity risk, borrowing constraints, and exogenous demographic transition. Its key institutional feature is China's segmented pension system: urban employee participants are covered by a combined-accounts scheme with both individual-account and pooling components, while urban–rural resident participants and non-participants face separate contribution and benefit rules. The model tracks the pension-state variables needed to implement these statutory formulas and to measure the fiscal inertia embedded in existing pension claims. We describe the demographic block first, then the household, firm, and government sectors, and finally the equilibrium definition.

2.1 Demographics

Population dynamics in the model are taken from the UN *World Population Prospects* (2024 revision) projections for China. These projections embed both the secular decline in fertility and the imminent retirement of the 1962–1973 baby-boom cohorts, and provide the core exogenous driving force for the analysis.

Let $N_t(j)$ denote the population of age j at time t. Agents live from age 20 ($j = 0$) to at most age 99 ($j = 79$). Working-age individuals ($j \leq j_R$) participate in labor and pay social security contributions while simultaneously making consumption and savings decisions; retired individuals ($j > j_R$) receive pensions and continue making consumption and savings decisions, where $j_R = 40$ (age 60) under the baseline retirement policy.

In each period t, new cohorts of age $j = 0$ enter the economy. The size of the new cohort $N_t(0)$ is determined by the fertility behavior of the childbearing population. Specifically, let ψ_j denote the female share in the age-j cohort, and let $n_t(j)$ denote the age-specific fertility rate for age-j women at time t. Then the new cohort size satisfies:

$$N_t(0) = \sum_{j \in F} \psi_j n_t(j) N_t(j) \tag{1}$$

where F denotes the set of reproductive age groups. The model treats fertility rates as exogenous parameters that can be influenced by policy, rather than as endogenously determined outcomes of household optimization. We consider three fertility scenarios: (i) Baseline (TFR $\approx$ 1.0), (ii) Replacement TFR (fertility scaled to TFR = 2.0), and (iii) High Fertility (baseline $\times$ 1.2).

Individuals face mortality risk captured by the age-dependent survival probability $\alpha_t(j)$, which represents the probability that an age-j individual survives to age $j + 1$. The intergenerational population evolution follows:

$$N_{t+1}(j + 1) = \alpha_t(j) N_t(j) \tag{2}$$

All population sequences are projected for 51 years (2023–2073), then frozen at the year-50 distribution for the remainder of the transition horizon ($T = 200$ periods). This freeze assumption ensures that the terminal steady state has a well-defined stationary population structure to which the economy can converge.

2.2 Household Sector

Each individual enters the economy at the beginning of adulthood ($j = 0$) and lives until the maximum age $J = 80$ (age 100). Individuals face mandatory retirement at age j_R.

Population types. The economy features three top-level pension-participation types, each (where applicable) decomposed by urban/rural location, with population shares calibrated from 2023 MoHRSS administrative data (Table 6):

- **UEBPI participants** ($\tau = U$, always urban formal): enrolled in the Urban Employee Basic Pension Insurance with full combined-accounts benefits;
- **URRBP participants** ($\tau = R$, with location superscript $\ell \in \{u, r\}$): enrolled in the Urban–Rural Resident Basic Pension;
- **Non-participants** ($\tau = N$, with location superscript $\ell \in \{u, r\}$): not enrolled in any pension system.

Population shares $\phi^{\tau,\ell}$ satisfy $\phi_U + \sum_\ell(\phi_R^\ell + \phi_N^\ell) = 1$ (the (U, r) combination is empty by construction). UEBPI participants retain a 4-dimensional state $\Omega^U_{i,j,t} = (a_{i,j,t}, b_{i,j,t}, e_{i,j,t}, \gamma_{i,j,t})$, where a denotes tradeable assets, b the pension individual account balance, e the historical average wage, and γ the idiosyncratic labor productivity shock; the individual account balance b is defined only for them. URRBP participants and non-participants share a 3-dimensional state $\Omega^{\tau,\ell}_{i,j,t} = (a_{i,j,t}, e_{i,j,t}, \gamma_{i,j,t})$ (with $b \equiv 0$); the location superscript ℓ selects the wage-efficiency factor η^ℓ ($\eta^u = 1$, $\eta^r = \eta_r = 0.419$), reflecting observed urban–rural disposable-income differentials in the data. Aggregates are formed by weighting each type by its calibrated population share.

The individual utility function takes the CRRA form:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \tag{3}$$

The lifecycle objective for a type-τ individual ($\tau \in \{U, R, N\}$, with location superscript ℓ where applicable) maximizes:

$$V^{\tau,\ell}_{i,j,t}(\Omega^{\tau,\ell}_{i,j,t}) = \max_{c,\,a'} \left\{ u(c^{\tau,\ell}_{i,j,t}) + \beta\,\alpha_t(j)\,\mathbb{E}\Big[V^{\tau,\ell}_{i,j+1,t+1}(\Omega^{\tau,\ell}_{i,j+1,t+1})\Big] + \beta(1-\alpha_t(j))\,V^b(a^{\tau,\ell}_{i,j+1,t+1}) \right\} \tag{4}$$

where $V^b(a) = \psi_1 \frac{(\psi_2+a)^{1-\sigma}}{1-\sigma}$ is the warm-glow bequest utility.

For **UEBPI participants** ($\tau = U$, always urban formal), the budget constraint applies an additive payroll deduction with a capped contribution base:

$$\begin{aligned}(1+\tau_c)c^U_{i,j,t} + a^U_{i,j+1,t+1} = {} & (1 + r_t(1-\tau_a))a^U_{i,j,t} \\ & + \mathbf{1}_{\{j \le j_R\}} \left[(1-\tau_{w,t})\, w_t\, \ell(j)\, \gamma_{i,j,t} - \tau_b\, \widetilde{w}_{i,j,t}\right] \\ & + \mathbf{1}_{\{j > j_R\}}\, s^U_{i,j,t} + \mathbf{1}_{\{j_{beq} \le j \le \bar{j}_{beq}\}}\, beq_t \end{aligned} \tag{5}$$

subject to $a^U_{i,j+1,t+1} \ge 0$. Here $\ell(j)$ is the deterministic age-efficiency profile, $\gamma_{i,j,t}$ is the idiosyncratic productivity shock, τ_b is the UEBPI total contribution rate, and $\widetilde{w}_{i,j,t}$ is the labor income clipped to the statutory $[60\%, 300\%]$ band of the social average wage (equation (9) below).

For **URRBP participants and non-participants** ($\tau \in \{R, N\}$, with location $\ell \in \{u, r\}$), the budget constraint applies a multiplicative wedge to labor income with per-type contribution rate $\tau_{b,\tau}$:

$$\begin{aligned}(1+\tau_c)c^{\tau,\ell}_{i,j,t} + a^{\tau,\ell}_{i,j+1,t+1} = {} & (1 + r_t(1-\tau_a))a^{\tau,\ell}_{i,j,t} \\ & + \mathbf{1}_{\{j \le j_R\}}\, (1-\tau_{b,\tau})(1-\tau_{w,t})\, w_t\, \eta^\ell\, \ell(j)\, \gamma_{i,j,t} \\ & + \mathbf{1}_{\{j > j_R\}}\, s^\tau_{i,j,t} + \mathbf{1}_{\{j_{beq} \le j \le \bar{j}_{beq}\}}\, beq_t \end{aligned} \tag{6}$$

subject to $a^{\tau,\ell}_{i,j+1,t+1} \ge 0$, where $\tau_{b,R} = \tau^r_b$ and $\tau_{b,N} = 0$, and $s^N_{i,j,t} \equiv 0$ (non-participants receive no pension flow). UEBPI participants face an additive deduction $(1-\tau_{w,t})\, w_t \ell(j)\gamma_{i,j,t} - \tau_b\, \widetilde{w}_{i,j,t}$, while URRBP participants face a multiplicative wedge $(1-\tau^r_b)(1-\tau_{w,t})$, reflecting separate institutional arrangements. The wage-efficiency factor η^ℓ ($\eta^u = 1$, $\eta^r = \eta_r$) is the only source of cross-location wage variation.

The idiosyncratic labor productivity shock follows an AR(1) process:

$$\log(\gamma_{i,j+1,t+1}) = \theta \log(\gamma_{i,j,t}) + \nu_{i,j+1,t+1}, \qquad \nu_{i,j+1,t+1} \sim N(0, \sigma_\nu^2) \tag{7}$$

which is discretized into a three-state Markov chain $\gamma \in \{0.36, 1.0, 2.70\}$.

2.2.1 Urban–Rural Pension System

A salient feature of China's social security system is its segmented urban–rural structure, which the model explicitly incorporates.

UEBPI participants ($\tau = U$) are enrolled in the Urban Employee Basic Pension Insurance, which combines a pooling account with an individual account. For retirees ($j > j_R$), pension benefits are:

$$s_{i,j,t}^U = \underbrace{\frac{b_{i,j,t}}{M(j_R)/12}}_{\text{Individual account}} + \underbrace{\alpha_b \cdot e_{i,j,t} \cdot j_R}_{\text{Pooling account}} \qquad (j > j_R) \tag{8}$$

where $M(j_R)$ is the statutory annuity period in months for the real retirement age implied by the model retirement index j_R, taken from the State Council schedule (State Council Document [2005] No. 38, Personal-Account Pension Annuity Months Schedule; reproduced in Table A.8 of the appendix), α_b is the pooling benefit rate, and $e_{i,j,t}$ is the individual's historical average wage. The annuity divisor $M(j_R)/12$ shrinks gradually with the retirement age (from 11.6 years at age 60 to 4.7 years at age 70), so per-period benefits rise modestly with delayed retirement; the pooling multiplier j_R should be read as the number of contribution years implied by that retirement age, rather than as the real age itself. This structure creates two opposing mechanical effects of delayed retirement: it raises per-period benefits through a smaller annuity divisor and a larger contribution-years multiplier, while reducing the number of years over which benefits are paid and increasing contribution years.

Contribution base bounds. Following statutory UEBPI rules, the wage that enters contributions, the personal-account accumulation b, and the historical-wage state e is clipped between 60% and 300% of the local previous-year social average wage $\bar{W}_t$:

$$\widetilde{w}_{i,j,t} \;\equiv\; \text{clip}\big(w_t \cdot \ell(j) \cdot \gamma_{i,j,t},\ 0.6\,\bar{W}_t,\ 3\,\bar{W}_t\big). \tag{9}$$

Throughout the model: contributions are $\tau_b \widetilde{w}$ (deducted from after-tax labor income, so worker take-home is $(1-\tau_w)\, w_t \ell(j) \gamma_{i,j,t} - \tau_b \widetilde{w}$); the personal account accumulates with $\tau^{bw} \widetilde{w}$; the wage index evolves with $e_{i,j+1,t+1} = \frac{j\,(\bar{W}_t/\bar{W}_{t-1})\, e_{i,j,t} + \widetilde{w}_{i,j,t}}{j+1}$. At the calibrated initial condition the 60% floor binds for low-productivity workers (γ near the bottom of the discretised grid), raising their effective contribution base above market labor income. The cap is real China policy and matters for the indexed-wage state e that feeds the pooling pension $\alpha_b\, e\, j_R$.

The individual account accumulates during working years at a fixed pension interest rate r_{ss}:

$$b_{i,j+1,t+1} = \begin{cases} (1 + r_{ss})\, b_{i,j,t} + \tau^{bw}\, \widetilde{w}_{i,j,t} & \text{if } j \le j_R \\ b_{i,j,t} & \text{if } j > j_R \end{cases} \tag{10}$$

where τ^{bw} is the personal account contribution rate and r_{ss} is the administratively set pension account interest rate (distinct from the market rate r_t). After retirement, b is frozen at its retirement value $b_{i,j_R,t}$ and is annuitized through the statutory divisor in equation (8); the personal account balance does not deplete as a state variable in our setup, consistent with the institutional treatment in which the account is administratively annuitized rather than literally drawn down.

The historical average wage evolution incorporates a *social average wage* mechanism. Define the social average wage as

$$\bar{W}_t = \frac{w_t\, L_t^U}{N_t^{\text{workers},U}} \tag{11}$$

where L_t^U is the aggregate effective labor of UEBPI participants (Type U) and $N_t^{\text{workers},U}$ is the number of UEBPI participants of working age. The historical average wage updates as:

$$e_{i,j+1,t+1} = \begin{cases} \dfrac{j \cdot \dfrac{\bar{W}_t}{\bar{W}_{t-1}} \cdot e_{i,j,t} + \widetilde{w}_{i,j,t}}{j+1} & \text{if } j < j_R \\ \dfrac{\bar{W}_t + \tilde{e}_{i,j,t}}{2} & \text{if } j = j_R \\ e_{i,j,t} & \text{if } j > j_R \end{cases} \tag{12}$$

where $\tilde{e}_{i,j,t}$ denotes the working-period update evaluated at age $j = j_R$. The ratio $\bar{W}_t/\bar{W}_{t-1}$ rescales the accumulated average wage to reflect economy-wide wage growth, ensuring that individuals who worked in lower-wage periods are not penalized at retirement; in steady state this ratio equals one and the formula reduces to the standard cumulative average. At the exact retirement age $j = j_R$, the accumulated average wage is converted into a *deemed contribution index* by averaging it with the current social average wage $\bar{W}_t$, following the institutional rules of China's UEBPI. After retirement ($j > j_R$), the index is frozen.

URRBP participants ($\tau = R$, location $\ell \in \{u, r\}$) are enrolled in the Urban–Rural Resident Basic Pension; we model their benefits as a simple replacement-rate benefit tied to historical average earnings:

$$s^R_{i,j,t} = \rho_r \cdot e_{i,j,t} \qquad (j > j_R) \tag{13}$$

where ρ_r is the replacement rate. URRBP participants do not have individual pension accounts ($b \equiv 0$), reflecting the simpler benefit structure of the URRBP. Their historical average wage follows a two-case rule without social average wage indexation:

$$e^{R,\ell}_{i,j+1,t+1} = \begin{cases} \dfrac{j \cdot e^{R,\ell}_{i,j,t} + w_t\, \eta^\ell\, \ell(j)\, \gamma_{i,j,t}}{j+1} & \text{if } j \le j_R \\ e^{R,\ell}_{i,j,t} & \text{if } j > j_R \end{cases} \tag{14}$$

There is no wage-growth rescaling ($\bar{W}_t/\bar{W}_{t-1}$) and no deemed-contribution-index conversion at retirement, since the URRBP does not employ these mechanisms. The wage-efficiency factor η^ℓ ($\eta^u = 1$, $\eta^r = \eta_r$) differentiates urban and rural URRBP participants.

Non-participants ($\tau = N$, location $\ell \in \{u, r\}$) are not enrolled in any pension system. They make no social-security contribution ($\tau_{b,N} = 0$) and receive no pension flow ($s^N_{i,j,t} \equiv 0$); their wage history e is therefore irrelevant for fiscal flows. Like URRBP participants, they have no individual account ($b \equiv 0$) and their labor income is scaled by the location-dependent efficiency factor η^ℓ.

2.3 Firm Sector

A representative firm operates a Cobb–Douglas production function:

$$Y_t = Z K_t^\alpha L_t^{1-\alpha} \tag{15}$$

where Z is TFP and α is the capital share. Profit maximization yields factor prices:

$$r_t = \alpha Z K_t^{\alpha-1} L_t^{1-\alpha} - \delta, \qquad w_t = (1-\alpha) Z K_t^\alpha L_t^{-\alpha} \tag{16}$$

Capital market clearing requires $K_t = A_t - D_t$, where A_t is aggregate household assets and D_t is government debt. The resource constraint is:

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t + G_t \tag{17}$$

2.4 Aggregation

Let $\mu^{\tau,\ell}_{j,t}(\Omega)$ denote the population-weighted measure of age-j individuals of type $\tau \in \{U, R, N\}$ in location $\ell \in \{u, r\}$ at time t, where the (U, r) combination is empty by construction. Aggregate quantities are obtained by summing over types and integrating over the joint distribution of states:

$$A_t = \sum_{j=0}^{J-1} \sum_{\tau,\ell} \int a_{i,j,t}\, d\mu_{j,t}^{\tau,\ell} \tag{18}$$

$$C_t = \sum_{j=0}^{J-1} \sum_{\tau,\ell} \int c_{i,j,t}\, d\mu_{j,t}^{\tau,\ell} \tag{19}$$

$$L_t = \sum_{j=0}^{j_R} \sum_{\tau,\ell} \int \eta^{\ell}\, \ell(j)\, \gamma_{i,j,t}\, d\mu_{j,t}^{\tau,\ell} \tag{20}$$

$$S_t = \sum_{j=j_R+1}^{J-1} \sum_{\tau,\ell} \int s_{i,j,t}^{\tau}\, d\mu_{j,t}^{\tau,\ell} \tag{21}$$

$$B_t = \sum_{j=0}^{j_R} \sum_{\tau,\ell} \int \tau_{b,\tau}\, \widetilde{w}_{i,j,t}^{\tau,\ell}\, d\mu_{j,t}^{\tau,\ell} \tag{22}$$

$$Beq_t = \sum_{j=0}^{J-1} (1-\alpha_t(j)) \sum_{\tau,\ell} \int a'_{i,j,t}\, d\mu_{j,t}^{\tau,\ell} \tag{23}$$

The type-specific objects entering pension expenditure and contribution revenue are summarized in Table 1.

Table 1: Per-type aggregation coefficients

τ	Pension flow s^{τ}	Contribution factor	Contribution wage $\widetilde{w}^{\tau,\ell}$
U	$\dfrac{b}{M(j_R)/12} + \alpha_b\, e\, j_R$	τ_b	$\text{clip}\big(w_t\, \ell(j)\, \gamma,\ 0.6\, \bar{W}_t,\ 3\, \bar{W}_t\big)$
R	$\rho_r\, e$	$\tau_b^r\, (1-\tau_{w,t})$	$w_t\, \eta^{\ell}\, \ell(j)\, \gamma$
N	0	0	(no contribution)

For UEBPI participants ($\tau = U$, always urban formal so $\ell = u$), the contribution wage is the statutorily capped $\widetilde{w}_{i,j,t}$ defined in equation (9); this is consistent with the household-level contribution $\tau_b\, \widetilde{w}$ in equation (5). For URRBP participants and non-participants, the contribution wage is uncapped but scaled by the location-dependent efficiency factor η^{ℓ}, with $\tau_{b,R}\, w\eta^{\ell}\ell(j)\gamma$ collapsing to the household's after-tax labor-income deduction in equation (6) once the multiplicative wedge is unpacked. The distributions $\mu_{j,t}^{\tau,\ell}$ evolve forward according to the household decision rules, the exogenous Markov process for γ, and the calibrated population shares $\phi^{\tau,\ell}$.

2.5 Government Sector

The government budget constraint is:

$$G_t + (1 + r_t)D_t + S_t = D_{t+1} + T_t + B_t \tag{24}$$

where G_t is government spending, S_t is total pension expenditure, $T_t = \tau_w w_t L_t + \tau_a r_t A_t + \tau_c C_t$ is total tax revenue (the bequest tax rate $\tau_{beq} = 0$ in the current calibration), and B_t is total social security contributions as defined in equation (22).

Accidental bequests left by deceased individuals are pooled and redistributed each period as an equal lump-sum transfer beq_t to all surviving agents in the age range $[j_{beq}, \bar{j}_{beq}] = [0, 20]$ (real ages 20–40), independent of urban/rural status. The gross bequest pool is redistributed in full.[1]

2.5.1 Fiscal Closure

Government spending is fixed at its 2023 share $G/Y = 21.8\%$ and the target debt ratio at $D/Y = 23.8\%$. The labor income tax rate τ^w adjusts to satisfy the government budget constraint in steady state:

$$\tau^w = \frac{G + rD + S - \tau_a r A - \tau_c C - B}{wL}. \tag{25}$$

Along the transition path, τ^w is set to a constant transitional rate τ^w_{trans} during the demographic transition phase, then switches to the terminal steady-state value; the scalar τ^w_{trans} is determined by bisection so that accumulated debt reaches the target D/Y at the end of

[1] We set $\tau_{beq} = 0$ in the baseline calibration.

the transition.[2] This closure answers the question: *How much must taxes rise to maintain fiscal sustainability?*

As a sensitivity benchmark, Online Appendix A.5 reports an alternative, less-distortionary closure in which all tax rates are held at their 2023 values and government spending G_t adjusts residually,

$$G_t = D_{t+1} + T_t + B_t - (1 + r_t)D_t - S_t, \tag{26}$$

so that G/Y absorbs the fiscal residual of population aging.

2.6 Equilibrium Definition

A competitive equilibrium along the transition path consists of household decision rules $\{c_t(\Omega), a'_t(\Omega)\}$, factor prices $\{w_t, r_t\}$, distributions $\{\mu^j_t\}$, and aggregates $\{K_t, L_t, Y_t, C_t\}$ such that: (1) households optimize given prices and policies; (2) firms optimize; (3) labor and capital markets clear; (4) the government budget balances each period; (5) bequests clear; and (6) the resource constraint holds.

The transition path is solved by iterating a backward EGM sweep over household problems with a forward Monte Carlo simulation of 800,000 agents, updating $\{r_t, w_t\}$ via damped fixed-point iteration; under τ^w-adjustment a bisection on a constant transitional wage tax τ^w_{trans} enforces the terminal debt-to-GDP target. Algorithms 1 and 2 in the Online Appendix give the full procedure; Appendix A.1 derives the EGM step in marginal-value form.

3 Calibration

This section describes how the model is disciplined by external evidence and targeted moments. Tables 2–7 summarize the externally calibrated parameters; Table 8 reports the internally calibrated targets. The calibration first assigns parameters from external sources

[2] This constant-rate transition convention follows Huang et al. [1997] and De Nardi et al. [1999]; see Algorithm 1 in the Appendix for details.

where possible, then jointly chooses the remaining parameters to match 2023 moments and validates the resulting model against the realized 2024 pension deficit.

3.1 Demographics

The model period is one year. Individuals enter the economy at age 20 ($j = 0$) and can live to a maximum of age 100 ($j = 79$). The baseline retirement age is 60 ($j_R = 40$). Survival probabilities are calibrated following İmrohoroğlu and Zhao [2018]. Age-specific fertility rates are calibrated to the UN *World Population Prospects* (2024 revision) for China's 2023 population. Population sequences are projected for 51 years (2023–2073) and then frozen at the year-50 distribution. Figure 1 shows the resulting trajectories: under baseline fertility (TFR $\approx$ 1.0), the terminal population falls to 60% of its initial level and the old-age dependency ratio rises from 33.7% to 71.1%; under replacement fertility (TFR = 2.0), the terminal population is 83.9% with a dependency ratio of 42.3%.

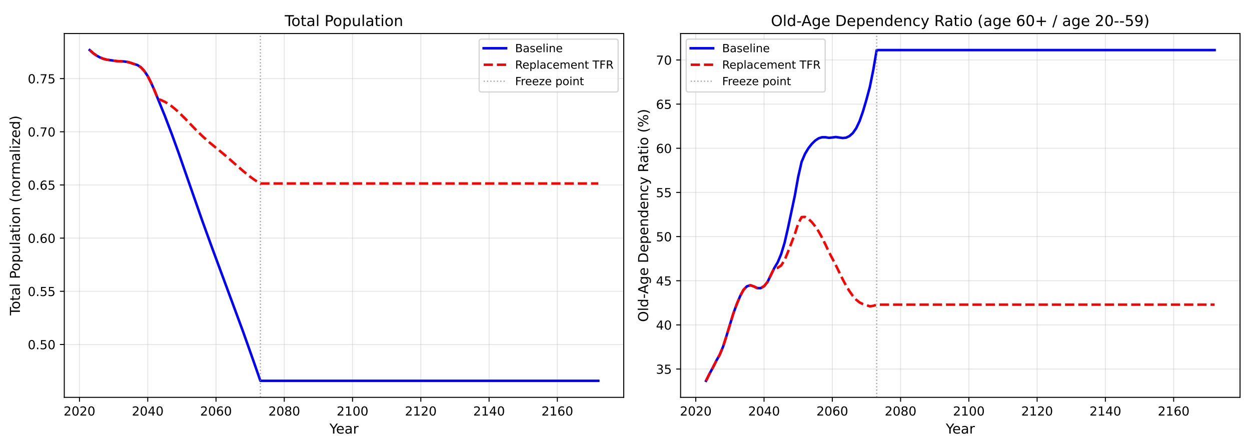


Figure 1: Population Trajectories and Dependency Ratios by Scenario

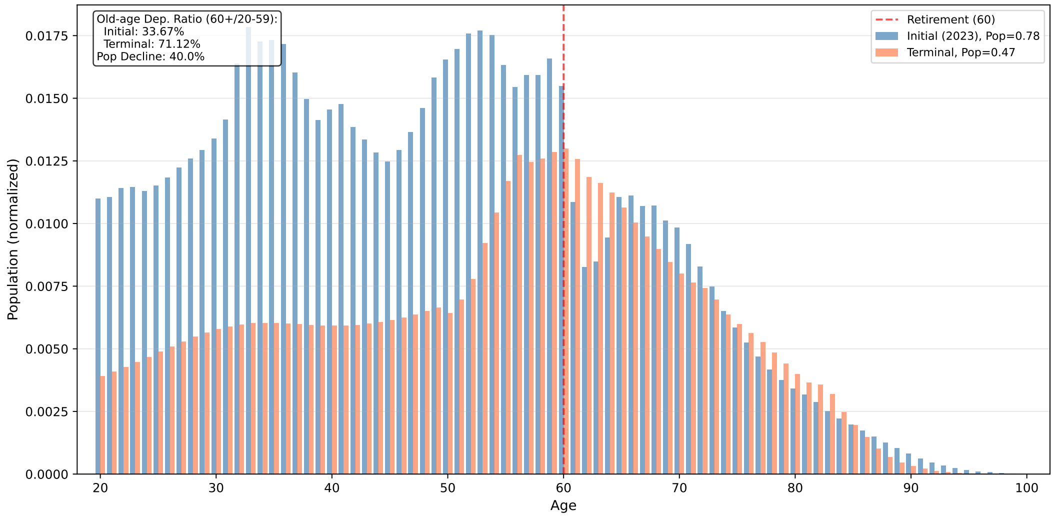


Figure 2: Population age distribution: baseline scenario, 2023 initial condition vs. terminal steady state. The age profile shifts sharply from a bottom-heavy pyramid in 2023 to a top-heavy distribution in the terminal state, reflecting the combined effect of declining fertility and rising life expectancy embedded in the UN projections. The inset reports the old-age dependency ratio (population 60+ relative to working-age 20–59) for each snapshot.

Table 2: Demographic Parameters

Parameter	Symbol	Value
Max model age	J	80 (age 100)
Retirement age	j_R	40 (age 60)
Transition periods	T	200

3.2 Preferences and Income Process

The risk aversion coefficient σ is set to 2, and the discount factor β is set to 0.99, both standard values in the Chinese-OLG literature (e.g., İmrohoroğlu and Zhao [2018, 2020]). The bequest weight ψ_1 is calibrated internally (Section 3.5); the bequest curvature ψ_2 is held fixed at 1.0 in model units—approximately one year of per-capita consumption at the initial condition—following the De Nardi [2004] bequest-as-luxury normalization, which places the bequest motive in the regime where low-wealth households rationally leave near-zero bequests. Their values are reported in Table 3. The income process follows $\log(\gamma') = 0.86\log(\gamma) + \nu$, $\sigma_\nu^2 = 0.06$, based on Yu and Zhu [2013], discretized to three states $\gamma \in$

$\{0.36, 1.0, 2.70\}$ following İmrohoroğlu and Zhao [2020]. Urban workers have an age-efficiency profile that peaks around age 50 (see the Online Appendix, Figure A.10); rural workers have labor efficiency $\eta_r = 41.9\%$ of urban workers, reflecting observed wage differentials. The five population shares (UEBPI, urban-informal-URRBP, urban-informal-no-system, rural-URRBP, rural-no-system) are computed from MoHRSS 2023 administrative records and reported in Table 6.

Table 3: Preference Parameters

Parameter	Symbol	Value
Discount factor	β	0.9900
Risk aversion	σ	2.00
Bequest weight	ψ_1	5.732
Bequest curvature	ψ_2	1.000

3.3 Production

Following Li [2024], the capital share α is set to 0.5 and the depreciation rate δ to 0.04. TFP Z is held fixed at 0.55. Because Z enters as a level normalization of model output, all ratio-to-GDP quantities reported below (K/Y, G/Y, D/Y, S/Y, and the social-security deficit $(S-B)/Y$) are invariant to the choice of Z given the calibration targets; we fix Z at this value to break the near-collinearity between Z and the bequest weight ψ_1 in identifying the capital–output ratio. Table 4 reports these externally calibrated production parameters.

Table 4: Production Parameters

Parameter	Symbol	Value
TFP	Z	0.5500
Capital share	α	0.50
Depreciation rate	δ	0.04

3.4 Social Security and Fiscal Policy

The total SS contribution rate is $\tau_b = 24\%$, of which $\tau^{bw} = 8\%$ enters the individual account. The statutory UEBPI rate combines an employer-side payment (∼16%) and a worker-side payment (∼8%); in the household budget constraint we apply the full τ_b to labor income as a worker deduction, which corresponds to assuming full pass-through of the employer contribution to the wage. This is a standard simplifying assumption in the literature [İmrohoroğlu and Zhao, 2018, Gai et al., 2025]; with a competitive labor market and inelastic firm labor demand at the calibrated capital–output ratio, the partial-vs.-full pass-through choice changes the level of w but leaves the aggregate fiscal flows that anchor our calibration unchanged. The pooling benefit rate $\alpha_b = 1\%$. The notional personal-account return r_{ss} is calibrated internally to match the urban pension expenditure ratio S_u/Y; the calibrated value is $r_{ss} = 2.91\%$. The contribution base is bounded by $[60\%, 300\%]$ of the social average wage $\bar{W}$ (equation 9), following statutory rules. Rural URRBP workers' replacement rate is fixed at $\rho_r = 10\%$; their contribution rate τ_b^r is calibrated internally to match the rural contribution ratio B_r/Y, with calibrated value $\tau_b^r = 1.14\%$.[3]

Table 5: Social Security Parameters

Parameter	Symbol	Value
Panel A: Urban Employee Basic Pension Insurance (UEBPI)		
Total SS contribution rate	τ_b	24%
Personal account contribution rate	τ^{bw}	8%
Pooling benefit rate	α_b	1.0%
Account interest rate	r_{ss}	2.91%
Contribution base bounds	$[\underline{\kappa}, \overline{\kappa}]\bar{W}$	[60%, 300%]
Panel B: Urban-Rural Resident Basic Pension (URRBP)		
Replacement rate	ρ_r	10.0%
Contribution rate	τ_b^r	1.14%
Panel C: Heterogeneity (see Table 6 for population shares)		
Rural efficiency (relative to urban)	η_r	41.9%

[3]The model treats URRBP contributions as proportional to wage income, whereas the actual scheme is largely flat-rate per participant; the calibrated value of τ_b^r absorbs this functional-form difference. We adopt the wage-proportional form for consistency with the household budget constraint.

The five population types and their participation parameters are summarized in Table 6.

Table 6: Population Types and Pension-System Participation (2023 MoHRSS)

Type	Description	Fraction	η_{eff}	$\tau_b^{r,\text{eff}}$	ρ_r^{eff}
F	Formal — UEBPI	27.10%	1.000	UEBPI	UEBPI
UR	Urban Resident — URRBP	9.27%	1.000	0.0114	0.1000 e
UI	Urban Informal — no coverage	24.49%	1.000	0	0
RR	Rural Resident — URRBP	10.75%	0.419	0.0114	0.1000 e
RI	Rural Informal — no coverage	28.39%	0.419	0	0

Following Lü et al. [2024], fiscal parameters are set as shown in Table 7.

Table 7: Fiscal Policy Parameters

Parameter	Symbol	Value
Labor income tax	τ_w	9.6%
Asset income tax	τ_a	29%
Consumption tax	τ_c	13.3%
Initial debt-output ratio	D/Y	23.8%
Government spending-output	G/Y	21.8%

3.5 Targeted Moments and Out-of-Sample Validation

Five free parameters are calibrated jointly by Nelder–Mead to match five 2023 moments. The free parameters are the bequest weight ψ_1, the UEBPI population coverage ϕ_{UEBPI}, the URRBP population coverage ϕ_{URRBP}, the personal-account return r_{ss}, and the rural URRBP contribution rate τ_b^r. The targets are: (i) the equilibrium interest rate $r = 5\%$ (equivalently $K/Y = 5.556$ via the firm FOC), (ii) urban contribution and benefit ratios B_u/Y and S_u/Y, and (iii) rural contribution and benefit ratios B_r/Y and S_r/Y.

The identification is transparent: ψ_1 pins down aggregate savings and hence r; ϕ_{UEBPI} scales urban contributions B_u/Y; r_{ss} moves urban personal-account benefits and hence S_u/Y; ϕ_{URRBP} scales the rural contribution and benefit base; and τ_b^r pins down the rural contribution rate. The system is just-identified, and Table 8 shows that the model matches all targeted moments to within 0.0001 percentage points. Tables 3, 6, and 5 report the corresponding calibrated parameter values.

Table 8: Model vs. Data Calibration Targets

Moment	Target	Model
Interest rate r	5.00%	5.00%
($\Leftrightarrow K/Y$)	5.556	5.555
Urban contribution B_u/Y	4.467%	4.467%
Urban expenditure S_u/Y	5.020%	5.020%
Rural contribution B_r/Y	0.0920%	0.0920%
Rural expenditure S_r/Y	0.2672%	0.2672%

Out-of-sample validation. The 2023 fit above is matched by construction. The more informative check is therefore out of sample: whether the calibrated 2023 model predicts the 2024 pension deficit. Holding all 2023 policy parameters, prices, and the social average wage fixed at their 2023 stationary-equilibrium values, we advance the 2023 cross-sectional distribution by a single partial-equilibrium step and aggregate fiscal outcomes with the 2024 vintage population. Crucially, no 2024 fiscal outcome enters the calibration; the only 2024 input is the cohort-size vector from the UN *World Population Prospects*. The model predicts a 2024 pension deficit of -0.803% of GDP, close to the realized -0.804% (Ministry of Finance 2024 budget-execution report; Table 9). Re-solving the model in general equilibrium at 2024 demographics moves the predicted deficit only marginally, to -0.810%, indicating that the GE feedback over a single year is small relative to the direct demographic channel. The validation establishes that the model's mapping from demographic structure to pension flows—the key channel through which the long-run projections in Section 4 operate—reproduces realized fiscal data.

Table 9: Out-of-Sample Validation: 2024 Pension Deficit

Year	Model	Data	Gap (bp/GDP)
2023 (calibrated)	-0.728%	-0.728%	-0.0
2024 *(PE 1-period)*	-0.803%	-0.804%	+0.1
2024 *(GE re-solve)*	-0.810%	-0.804%	-0.6

Note: The 2024 row is rebased to the 2023 GDP / output base to remove the denominator effect of GDP growth (the stationary model abstracts from secular GDP growth). On the data side, the 2024 deficit in CNY is divided by 2023 GDP rather than 2024 GDP (a +7.02% nominal-GDP adjustment). On the model side, the 2024 deficit is divided by the model's 2023 Y_M.

4 Quantitative Analysis

This section presents the quantitative results of our model. We first report the initial condition (2023) and the terminal steady state (Sections 4.1–4.2), followed by the baseline transition path (Section 4.3). We then evaluate three policy reform experiments—delayed retirement, reduced pooling transfers, and replacement-level fertility—under the τ^w-adjustment closure (Sections 4.4–4.5), and report the distributional and intergenerational welfare effects of those reforms (Section 4.6). A complementary cost-effectiveness perspective on pro-natalist policy is developed separately in Section 5. Corresponding results under the G-adjustment sensitivity are reported in Online Appendix A.5.

4.1 Initial Condition (2023)

The starting point of the transition analysis is an *initial condition* calibrated to match key features of the Chinese economy in 2023: labor and capital markets clear, the government maintains constant policy parameters, and the household distribution is stationary given constant prices and policies, but the government budget does not balance—government debt is changing over time under the 2023 fiscal configuration. Table 10 reports the principal macroeconomic aggregates.

Table 10: Initial Condition: Macroeconomic Aggregates (2023)

Variable	Value	Variable	Value
Interest rate r	0.0500	Aggregate output Y	1.3637
Wage w	0.8402	Aggregate consumption C	0.7668
Aggregate capital K	7.5753	Capital–output ratio K/Y	5.56
Effective labor L	0.8115	Debt–output ratio D/Y	23.80%
SS revenue / Y	4.56%	SS expenditure / Y	5.29%
SS balance / Y	−0.73%	Gov. purchases G/Y	21.80%

The model generates a capital–output ratio of 5.556 matching the implied target exactly, corresponding to an equilibrium interest rate of 5.00%. The social security balance of −0.73% of GDP matches the 2023 data value of −0.73% exactly, with all five calibration moments hit to within 0.0001 pp (Table 8).

Government purchases amount to $G/Y = 21.80\%$ and the social security deficit is 0.73% of GDP. We take the debt ratio $D/Y = 23.8\%$ as exogenous at the initial date and require it to be maintained (or reduced) along the transition path. This modeling choice allows us to cleanly decompose the total fiscal burden into two components: a *debt burden*—the cost of servicing the inherited $D/Y = 23.8\%$—and an *aging burden*—the additional fiscal adjustment required by the demographic transition. As Section 4.2.2 shows, this decomposition reveals that population aging accounts for the dominant share of the total fiscal adjustment.

4.2 Terminal Steady States

The terminal steady state corresponds to the long-run equilibrium after the demographic transition is complete and the population distribution has stabilized. The main text reports the baseline τ^w-adjustment closure, while the alternative G-adjustment closure is reported as a sensitivity exercise in the Online Appendix.

4.2.1 τ^w-Adjustment Mode

Under the τ^w-adjustment rule, the wage tax rate adjusts each period to maintain D/Y at or below its cap of 23.80%, while government purchases as a share of GDP remain at their initial level of 21.80%. Table 11 reports the terminal aggregates.

Table 11: Terminal Steady-State Aggregates: τ^w-Adjustment

Variable	Value	Variable	Value
Wage tax τ^w	27.50%	Capital–output ratio K/Y	6.936
Interest rate r	0.0321	Debt–output ratio D/Y	23.84%
Wage w	1.0489	Gov. purchases G/Y	21.80%
SS revenue / Y	4.54%	SS expenditure / Y	10.98%
SS balance / Y	−6.44%		

The demographic transition imposes a substantial fiscal burden. The wage tax must rise from 9.60% to 27.50% to maintain the debt-to-GDP cap. Effective labor shrinks substantially as the working-age population contracts to 60% of its initial level. Despite the rise in the wage rate, the income base contracts so severely that the tax rate nearly triples. The social security deficit widens to 6.44% of GDP, reflecting the surge in pension expenditure relative to the shrunken contribution base. The capital–output ratio rises from 5.56 to 6.94 as labor scarcity raises the marginal product of capital, while the higher wage tax tempers but does not reverse the saving response. The corresponding terminal steady state under the alternative G-adjustment closure is reported in Online Appendix A.5.

4.2.2 Fiscal Burden Decomposition: Debt vs. Aging

Table 12 decomposes the total fiscal adjustment between the initial condition and the terminal steady state into a *debt burden*—the adjustment required to close the 2023 budget gap at 2023 demographics while maintaining $D/Y = 23.8\%$—and an *aging burden*, the residual adjustment driven by the demographic transition. Under the τ^w-adjustment closure, the aging burden accounts for 11.63 pp of the 17.90-pp total wage-tax increase. The dominant share of the long-run fiscal adjustment is driven by demographic aging rather than by inherited debt. The corresponding decomposition under the G-adjustment sensitivity is reported in Online Appendix A.5.

Table 12: Fiscal Burden Decomposition: Debt vs. Aging

	τ_w-Adjustment
Initial (2023)	$\tau_w = 9.60\%$
Counterfactual (2023 demog., balanced)	$\tau_w = 15.87\%$
Terminal (aged demog.)	$\tau_w = 27.50\%$
Debt burden	+6.27pp
Aging burden	+11.63pp
Total adjustment	+17.90pp

Note: The "counterfactual" row gives the fiscal instrument level that would balance the government budget at 2023 demographics (i.e., with no population aging) while maintaining $D/Y = 23.8\%$. The gap between the initial setting and the counterfactual is the debt burden; the gap between the counterfactual and the terminal steady state is the aging burden.

4.3 Baseline Transition Paths

We solve for the general equilibrium transition path from the initial condition (2023) to the terminal steady state over a horizon of 200 periods (years). The transition period during which the population distribution evolves is 51 years; thereafter, the population is frozen at its year-50 distribution and the economy converges to the terminal steady state. Along the transition path, all markets clear and household decisions are optimal at every period.

4.3.1 Transition under τ^w-Adjustment

Figure 3 displays the transition paths of key macroeconomic variables under τ^w-adjustment.

The wage tax rate is set at a constant transitional rate of $\tau_w^{\text{trans}} = 19.64\%$ during the demographic transition phase, before switching to the terminal rate of 27.50%. The roughly

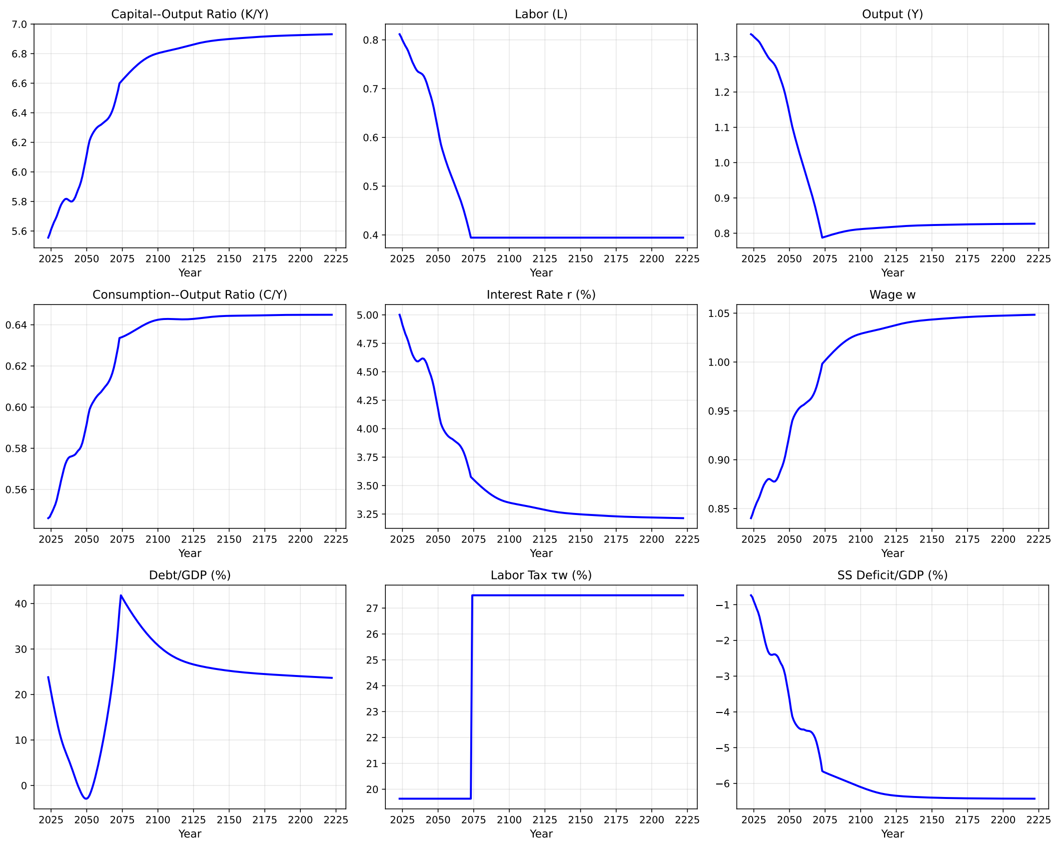


Figure 3: Transition paths: τ^w-adjustment mode

7.9-pp step reflects the fact that the bulk of the aging burden materialises in the post-transition period, when the largest cohorts have fully retired and the demographics have stabilised at the frozen distribution; during the transition phase there are still more workers per retiree, so a lower wage tax suffices to anchor the debt-to-GDP target. The transitional rate is determined by bisection to achieve the target D/Y at the end of the transition phase. Along the transition the wage rate rises and the interest rate declines, reflecting the increasing relative scarcity of labor as the working-age population contracts.

4.3.2 Urban–Rural Decomposition of Social Security Deficit

Because the pension systems for urban and rural workers differ fundamentally in generosity and structure, the aggregate social security deficit masks important compositional dynamics. We decompose the deficit $(S-B)/Y$ into its urban and rural components at each period along the transition path.

Figure 4 reports the decomposition; the underlying numbers are tabulated in the Online Appendix (Table A.10). Two features stand out. First, the urban pension system accounts for approximately 93% of the total deficit throughout the transition. The urban deficit rises from 0.55% of GDP in 2023 to 5.96% in the terminal steady state, driven by the generous combined-accounts benefit formula: as the urban retiree population swells, the pooling component ($\alpha_b \cdot \bar{e} \cdot j_R$) and personal account drawdowns together far outstrip contribution revenue. The rural deficit, by contrast, remains below 0.5% of GDP even at the end of the transition, reflecting the much lower calibrated rural replacement rate ($\rho_r = 10\%$) and smaller rural workforce.

Second, the widening deficit is driven almost entirely by the expenditure side. Urban pension expenditure S_u/Y roughly doubles over the transition horizon as the baby-boom cohorts retire and the retiree-to-worker ratio climbs, while urban contribution revenue B_u/Y remains relatively stable. Under current contribution rates, compliance, and coverage, the widening deficit is driven almost entirely by rising expenditure on the urban employee pension

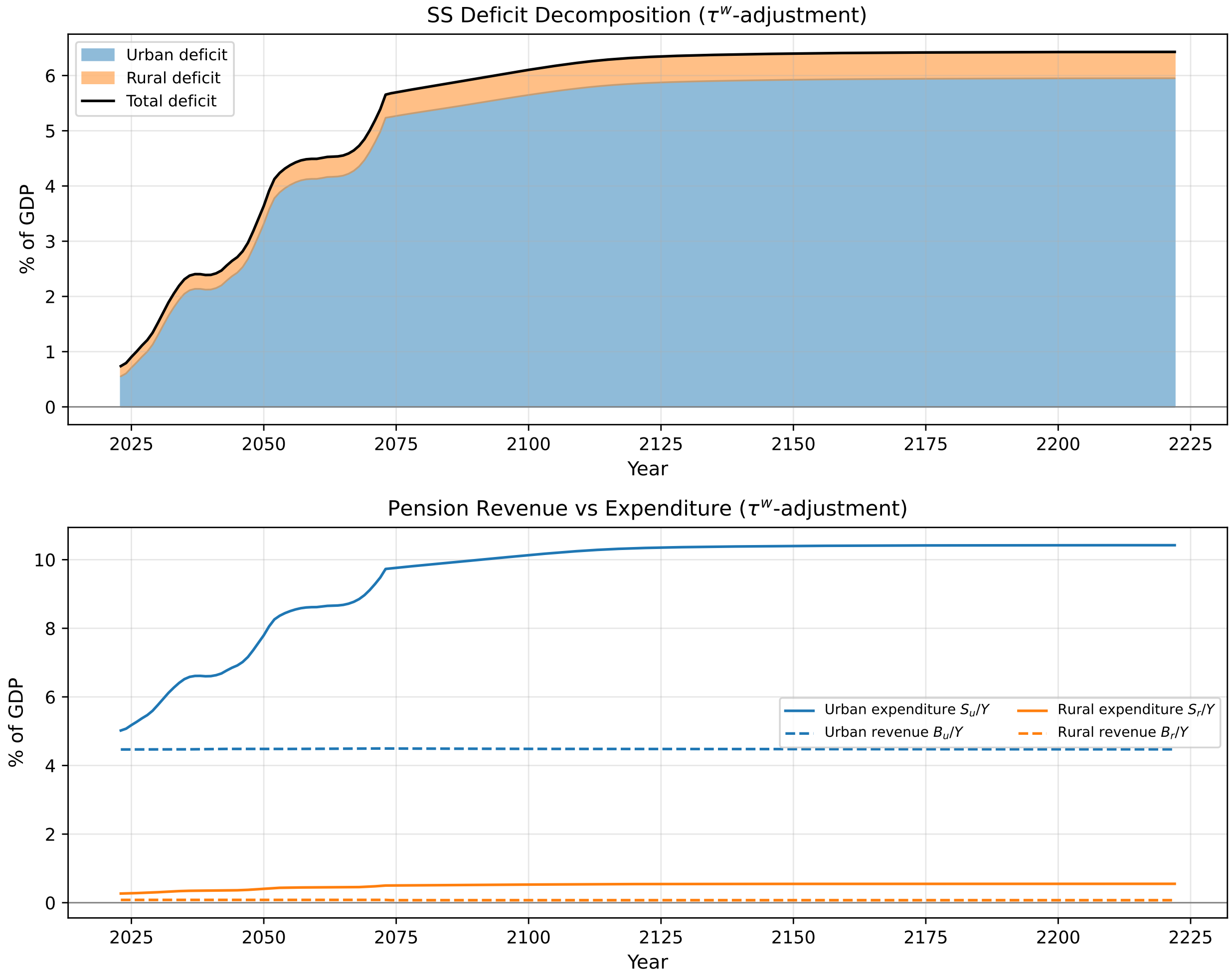


Figure 4: Decomposition of social security deficit into urban and rural components along the transition path under τ^w-adjustment. Top panels: stacked area showing the share of the total deficit attributable to each sector (% of GDP). Bottom panels: pension expenditure (S/Y) and contribution revenue (B/Y) by sector.

system.

4.4 Policy Experiment Design

Building on the baseline transition path, we evaluate three individual policy reform experiments and one combined experiment. Each individual experiment modifies one structural parameter while all other parameters remain at their baseline values. For each experiment, we solve the general equilibrium transition path under the τ^w-adjustment closure; the same experiments under the G-adjustment sensitivity are reported in Online Appendix A.5.

Experiment 1: Delayed Retirement ($j_R = 45$, i.e., age 65). The statutory retirement age is raised from 60 (model age 40) to 65 (model age 45). This extends the working life by five years, simultaneously increasing social security contributions and reducing the pension payment period. Because the pension formula uses the retirement-age index through the statutory annuity divisor and the implied number of contribution years (equation 8), per-period benefits increase under delayed retirement, partially offsetting the fiscal savings from fewer retirees.

Experiment 2: Reduced Pooling Transfers ($\alpha_b = 0.5\%$). The pooling benefit coefficient is halved from 1% to 0.5%, reducing the social-average-wage-based component of pension benefits. This weakens intragenerational redistribution and strengthens the actuarial link between contributions and benefits.

Experiment 3: Replacement-Level Fertility (TFR $= 2.0$). The total fertility rate is raised from the baseline of 1.0 to the replacement level of 2.0, treated as a counterfactual demographic path rather than as an endogenous fertility response to a specific policy package. The demographic effects materialize gradually: the new cohorts enter the labor market approximately 20 years after birth.

Experiment 4: Combined (Delayed Retirement + Replacement-Level Fertility). This experiment simultaneously raises the retirement age to 65 and the TFR to 2.0, capturing the interaction between supply-side and demographic reforms.

4.5 Policy Experiment Results: τ^w-Adjustment

Table 13 summarizes the policy experiment outcomes under τ^w-adjustment.

Table 13: Policy Experiment Summary: τ^w-Adjustment

Experiment	τ_w^{trans}	τ_w^{term}	$\Delta\tau_w^{trans}$	$\Delta\tau_w^{term}$
Baseline	19.63%	27.50%	—	—
Delayed Retirement (+5 years)	16.35%	22.40%	−3.3pp	−5.1pp
Reduced Pooling ($\alpha_b = 0.5\%$)	16.87%	23.62%	−2.8pp	−3.9pp
Replacement TFR (2.0)	18.89%	18.46%	−0.7pp	−9.0pp
Combined (Delayed Ret. + Repl. TFR)	15.92%	15.79%	−3.7pp	−11.7pp

Figure 5 plots the transition paths for all experiments under τ^w-adjustment.

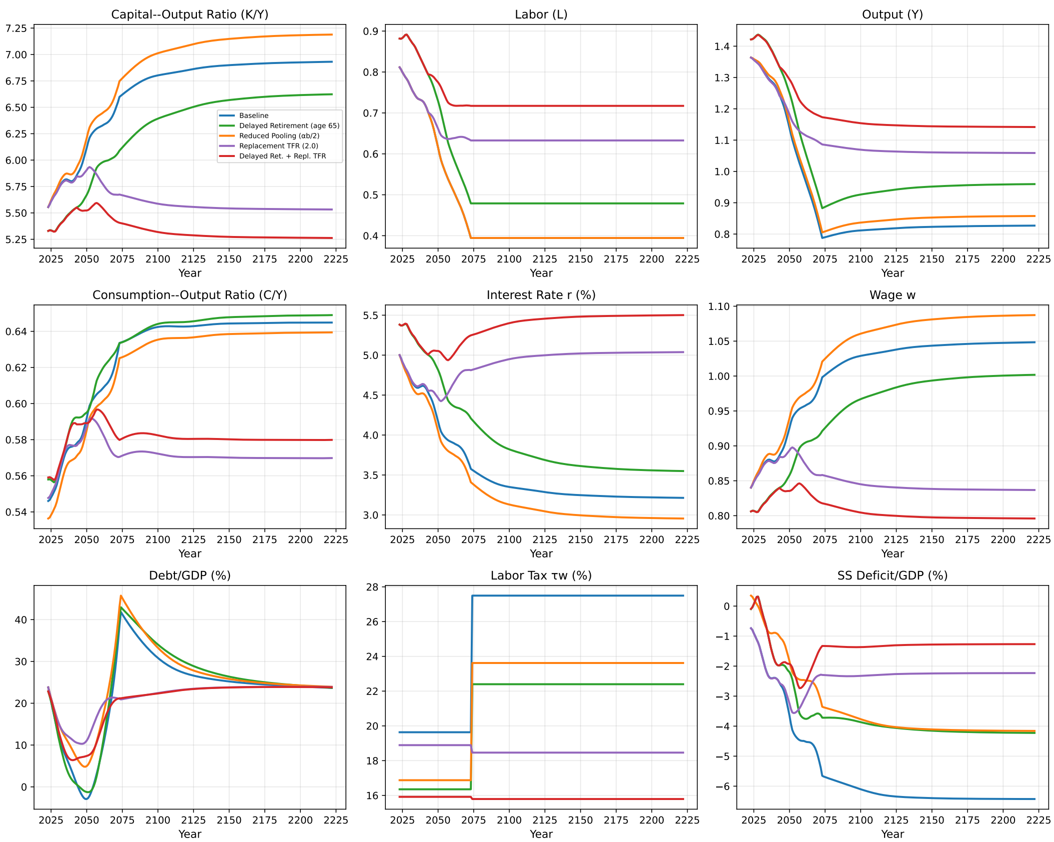


Figure 5: Policy experiment comparison: τ^w-adjustment mode

Reducing pooling transfers lowers the transitional rate by 2.8 percentage points (from 19.64% to 16.88%) and the terminal rate by 3.9 percentage points (from 27.50% to 23.63%).

This reform cuts pension expenditure on the pooling component of benefits with immediate effect, but it cannot eliminate the long-run gap.

Delayed retirement to age 65 lowers the transitional rate by 3.3 percentage points (to 16.35%) and the terminal rate by 5.1 percentage points (to 22.40%). The mechanism operates through two channels: extending the working life expands the contribution base, while the shorter retirement period reduces cumulative pension expenditure. Per-period benefits rise modestly under the statutory annuity formula—the divisor $M(j_R)/12$ falls from 11.6 years at $j_R = 40$ (age 60) to 8.4 years at $j_R = 45$ (age 65), and the pooling multiplier j_R rises proportionally—but this is small relative to the gains from a larger contribution base and shorter retirement. The terminal gain (−5.1 pp) exceeds the transitional gain (−3.3 pp) because in the long run the demographic stabilization combines with the extended contribution period to deliver the full relief, while during the transition only the cohorts at the edge of retirement benefit immediately. Further delaying retirement to age 70 yields a larger transitional effect (−4.6 pp) and a comparable terminal effect (−5.7 pp); the diminishing terminal return reflects the larger pooling multiplier j_R partially offsetting the demographic relief (Appendix Table A.13). These numbers treat retirement as the statutory mandatory event in our model, with no endogenous early-exit margin. The institutional context makes that an acceptable approximation in China: voluntary early retirement is rare under the existing UEBPI rules and labor-force participation rates remain high through the mandatory age in administrative data, so the full-compliance number is a reasonable benchmark rather than a strict upper bound.

Replacement-level fertility has a small effect on the transitional tax rate (−0.7 pp, to 18.90%) because the new cohorts have not yet entered the labor force during the peak fiscal pressure period. The terminal effect, however, is the largest of any single instrument: the terminal tax rate falls by 9.0 pp to 18.47% as the larger working-age population expands the contribution base and improves the dependency ratio. Combining delayed retirement to age 65 with replacement-level fertility yields the largest overall improvement, with the terminal rate falling 11.7 pp to 15.80%—the two instruments are complementary, not substitutes.

4.6 Distributional and Welfare Effects

Reform choice has a distributional dimension that the aggregate fiscal number cannot resolve. The individual-level structure of the model lets us trace, for each reform, which birth cohorts gain and which lose. The headline pattern is sharp: each single reform creates substantial intergenerational redistribution, while the combined delayed-retirement + fertility policy produces a cohort-CEV profile substantially flatter than either single reform across the cohort range. Distributional complementarity, in addition to fiscal complementarity, motivates the combined policy. Throughout this subsection, fiscal closure is the baseline τ^w-adjustment; corresponding results under the G-adjustment sensitivity are in Online Appendix A.5.

For each birth cohort t we compute a cohort-specific CEV

$$\lambda_t = \left(\frac{V_t^{\text{exp}} - V_{t,\text{beq}}^{\text{base}}}{V_{t,\text{flow}}^{\text{base}}} \right)^{1/(1-\sigma)} - 1, \tag{27}$$

where $V_t^{\text{exp}} = V_{t,\text{flow}}^{\text{exp}} + V_{t,\text{beq}}^{\text{exp}}$ is total expected discounted lifetime utility under the experiment transition, while $V_{t,\text{flow}}^{\text{base}}$ and $V_{t,\text{beq}}^{\text{base}}$ are the baseline transition's flow-consumption and warm-glow bequest components. The CEV scales only the baseline consumption stream, so the baseline bequest component is held fixed in the inversion. Figure 6 plots λ_t by birth year for each reform.

The mechanism is sharply distributional. Delayed retirement reshapes the pension benefit formula—through a smaller annuity divisor $M(j_R)/12$ and a larger pooling multiplier j_R—and these operate immediately on every retiree's predetermined account balance b and historical wage e; cohorts within roughly a decade of retirement absorb losses of up to -10 percentage points. Reduced pooling cuts the pooling component for all current retirees in proportion to their wage history. Higher fertility leaves current cohorts' pension state untouched and works only through the future contribution base; cohorts born in the first two

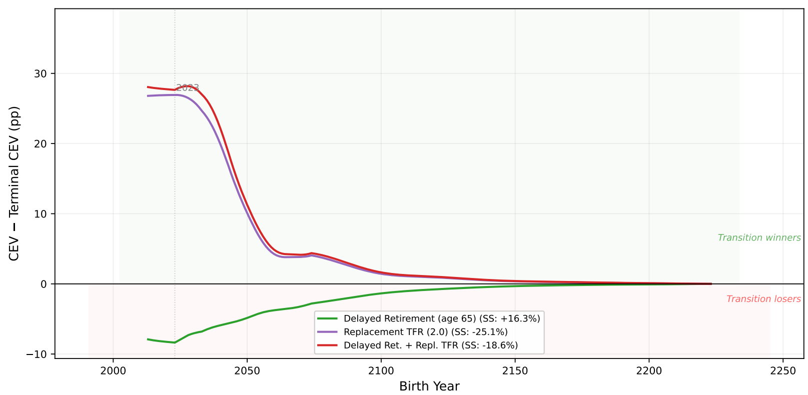


Figure 6: Cohort CEV by birth year for each reform relative to the baseline demographic transition under τ^w-adjustment, in percentage points and de-meaned by the terminal (last-cohort) CEV so that zero represents the long-run benchmark. Positive (negative) values identify cohorts that gain (lose) from each reform.

decades of the transition gain +11 to +14 percentage points by inheriting an improved dependency ratio without bearing the full contemporaneous tax burden. Combining delayed retirement with fertility pulls in opposite directions across the cohort distribution: the near-retiree losses from extended contribution years are partially offset for later cohorts by the dependency-ratio gain, smoothing the cohort-CEV profile substantially relative to either single reform. This intergenerational pattern is precisely the content that a uniform-transfer pension model cannot generate.

Cross-sectional inequality. Aggregate cross-sectional inequality measures tell a complementary story: the wealth Gini falls from 0.459 in 2023 to 0.403 in the terminal steady state (Table 14), but this compression is demographic (a shift toward an older, more homogeneous age structure) rather than redistributive—the underlying dispersion in pension claims is preserved and continues to drive the heterogeneous reform incidence shown above. The urban wealth Gini (0.453) exceeds the rural (0.440) at baseline, reflecting the richer heterogeneity embedded in the urban pension-account state; the top 10% of the wealth distribution holds 30.9% of total wealth and the bottom 50% holds 18.5%.

Table 14: Inequality: Steady-State Comparison (τ^w-adjustment)

	Initial (2023)	Terminal (τ^w-adj)
Gini coefficients		
Wealth	0.459	0.403
Urban	0.453	0.398
Rural	0.440	0.348
Consumption	0.309	0.252
Income	0.388	0.399
Wealth distribution		
P90/P10	16.3	9.8
Top 10% share	30.9%	26.8%
Bottom 50% share	18.5%	21.8%

Absolute welfare cross-check. As a final cross-check, the absolute terminal-SS CEVs (relative to the 2023 initial condition) are broadly comparable across urban and rural agents: +26.63% urban versus +27.92% rural (Table A.12 in the Online Appendix), with rural agents gaining slightly more (a 1.3-pp wedge that reflects the calibrated UEBPI footprint).[4]

5 Fiscal Value of Birth

We define the *fiscal value of birth* as the present-value fiscal gain per additional birth, discounted using transition-path equilibrium interest rates. The object answers a sharp question: given that each additional birth alleviates the future dependency burden, how large could a one-time per-birth subsidy be while still leaving the government no worse off in present-value terms?

The construction takes the additional births as given and reports what each induced birth is fiscally worth; whether a particular pro-natalist policy can actually generate those births—and at what cost—is outside the scope of this paper. It should therefore be read as a fiscal break-even benchmark rather than a full policy evaluation. It is not a welfare measure: it is silent on the household's valuation of additional children and on the behavioral

[4] Absolute CEV levels are sensitive to the omission of a direct utility value for government consumption G in the household problem; the urban–rural differential within each closure is robust to that caveat.

cost of inducing them. Because education, health, childcare, and other child-specific public expenditures are not netted out separately—only general government consumption G is netted in our calculation—the reported FV is an accounting upper bound under this fiscal convention.

5.1 Definition and Baseline Estimates

We report the fiscal value in two forms—a one-time lump-sum payment at birth (FV) and a constant annual payment over the first three years of life (FV^{ann}, matching the horizon of typical Chinese provincial child subsidies). Both forms equate the present value of the per-birth subsidy stream to the present value of the additional fiscal gain $\Delta\mathcal{F}_t$:

$$\boxed{\text{FV} = \frac{\sum_{t=0}^{T-1} \Delta\mathcal{F}_t / R_t}{\sum_{t=0}^{T-1} \Delta N_{0,t} / R_t}, \qquad \text{FV}^{\text{ann}} = \frac{\sum_{t=0}^{T-1} \Delta\mathcal{F}_t / R_t}{\sum_{t=0}^{T-1} \Delta N_{0,t} \cdot A_t}, \qquad A_t \equiv \sum_{k=0}^{2} \frac{1}{R_{t+k}}.} \tag{28}$$

Here $R_t \equiv \prod_{s=0}^{t-1}(1 + r_s)$ is the cumulative gross return, $\Delta N_{0,t} = \max(N_{0,t}^{\text{tfr}} - N_{0,t}^{\text{base}}, 0)$ is additional newborns, and A_t is the present value of a 3-year unit stream starting at t. Under G-adjustment the fiscal gain is $\Delta\mathcal{F}_t = G_t^{\text{tfr}} - G_t^{\text{base}}$ (additional spending the higher-fertility path can sustain); under τ^w-adjustment it is $\Delta\mathcal{F}_t = (\tau_t^{w,\text{base}} - \tau_t^{w,\text{tfr}}) \cdot w_t L_t$ (tax relief). Since $A_t > 1/R_t$, the annuity form is smaller than the lump-sum form—the same fiscal dividend spread over 3 years yields a smaller per-year amount. For births beyond the model horizon, R_{t+k} is extended using the terminal steady-state interest rate.

Model-unit values are converted to CNY using China's 2023 GDP of 126.06 trillion yuan and population of 1.41 billion.

We compute the fiscal value under two comparisons. First, "TFR vs Baseline" compares replacement-level fertility (TFR = 2.0) against the status quo (TFR = 1.0), measuring the pure effect of fertility on fiscal space. Second, "TFR + Retirement vs Retirement" compares the combined policy (TFR = 2.0 plus delayed retirement to age 65) against delayed retirement alone, isolating the marginal fiscal value of fertility when retirement reform is already in place.

Table 15: Fiscal Value of an Additional Birth

	TFR vs Baseline	TFR + Ret. vs Ret.
Lump-sum FV (10k CNY)	12.4	7.5
Annuity FV (10k CNY/yr, 3 yr)	4.3	2.6

Note: "TFR vs Baseline" compares replacement-level fertility (TFR = 2.0) against the status quo (TFR = 1.0). "TFR + Ret. vs Ret." compares the combined policy (TFR = 2.0 with delayed retirement to age 65) against delayed retirement alone, isolating the marginal fiscal value of fertility when retirement reform is already in place. Lump-sum FV is a one-time payment at birth; annuity FV is a constant annual payment for 3 years (ages 0–2). All values are discounted using transition-path equilibrium interest rates. All values in 2023 CNY (1 unit = 10,000 yuan).

Table 15 reports the per-birth fiscal value in both lump-sum and annuity form under the τ^w-adjustment closure. In the "TFR vs Baseline" comparison, the lump-sum fiscal gain per additional birth is 12.4 10k CNY (124,000 yuan), and the annuity-equivalent fiscal value over the first three years of life is 4.3 10k CNY/yr (43,000 yuan per year). The "TFR + Retirement vs Retirement" comparison yields a lower value—7.5 10k CNY lump-sum (2.6 10k CNY/yr annuity)—because delayed retirement already captures part of the fiscal improvement by extending contribution periods, so the marginal gain from additional births is smaller when retirement reform is already in place. Corresponding numbers under the G-adjustment sensitivity are reported in Online Appendix A.5.

5.2 Fiscal Returns at Intermediate Fertility Levels

The marginal fiscal value of an additional birth diminishes with fertility: it is *highest* at low fertility levels near China's current TFR of 1.0 and *declines* as fertility rises toward replacement. This counterintuitive pattern reflects the fact that the first additional births alleviate the most acute dependency pressure; even small, realistic fertility improvements can be fiscally justified if the cost of inducing each additional birth remains below its fiscal value, and replacement-level fertility is not required for a pro-natalist subsidy to break even in present value terms.

The preceding analysis assumed replacement-level fertility (TFR = 2.0) as a benchmark. In practice, China's current TFR of approximately 1.0 is unlikely to reach replacement level even with aggressive pro-natalist policies; a more realistic target may be TFR = 1.3–1.5. We therefore compute fiscal values across a range of hypothetical fertility levels from TFR = 1.1 to 2.0.

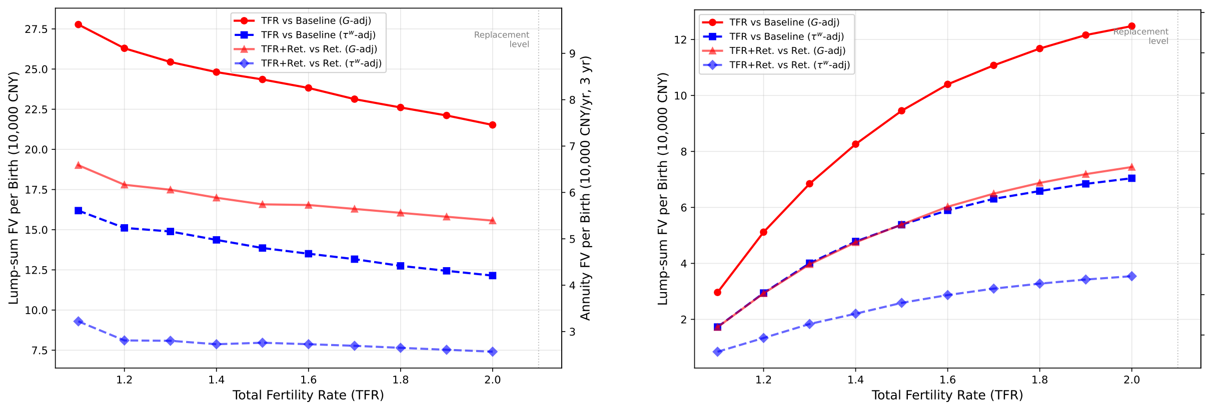
Table 16: Fiscal Value of Birth Across Fertility Levels (10k CNY)

	Lump-sum FV		Annuity FV (3 yr)	
TFR	Marginal	Universal	Marginal	Universal
1.1	16.2	1.7	5.61	0.60
1.3	14.9	4.0	5.16	1.39
1.5	13.9	5.4	4.80	1.87
1.8	12.7	6.6	4.42	2.28
2.0	12.1	7.0	4.20	2.44

Note: Marginal FV is the fiscal gain per additional birth; universal FV spreads the gain over all newborns. Lump-sum FV is a one-time payment at birth; annuity FV is a constant annual payment for 3 years (ages 0–2). All values are discounted using transition-path equilibrium interest rates, in 2023 CNY (1 unit = 10,000 yuan).

Table 16 and Figure 7 report the results under the τ^w-adjustment closure (corresponding numbers under the G-adjustment sensitivity are in Online Appendix A.5). The *marginal* fiscal value per additional birth *decreases* with TFR, from 16.2 10k CNY at TFR = 1.1 to 12.2 10k CNY at TFR = 2.0. This reflects diminishing marginal returns—the first additional births alleviate the most acute labor scarcity, while subsequent births face a crowded labor market and lower marginal productivity. Even modest fertility increases are therefore fiscally valuable, and the per-birth fiscal justification for subsidies is *strongest* at low fertility levels closest to the current situation.

By contrast, the *universal* fiscal value—the subsidy that can be justified per actual newborn (not just per additional birth)—*increases* with TFR, from 1.7 10k CNY at TFR = 1.1 to 7.1 10k CNY at TFR = 2.0. Larger fertility programs generate greater total fiscal gains spread over more births, yielding higher per-newborn subsidies. The marginal FV applies to a perfectly targeted subsidy paid only to induced births, whereas the universal FV applies when the subsidy is paid to all births, including inframarginal births. The universal FV is the lower bound on a fiscally-effective subsidy when policy cannot target the marginal household (i.e., the subsidy must be paid to every newborn); the marginal FV is the upper benchmark for a subsidy paid only to induced births.



(a) Marginal FV (per additional birth)

(b) Universal FV (per actual birth)

Figure 7: Fiscal value of birth as a function of target TFR. Left: marginal (per additional birth); right: universal (per actual birth under the policy). The left y-axis shows the lump-sum present value (10k CNY paid once at birth); the right y-axis shows the annuity-equivalent constant annual payment over 3 years (10k CNY/yr). The figure also displays the G-adjustment sensitivity (reported in Online Appendix A.5) for visual comparison; the τ^w-adjustment lines (with and without concurrent delayed retirement) are the focus of the main analysis.

In the policy-relevant range of TFR = 1.3–1.5, the marginal lump-sum fiscal value is 13.9–14.9 10k CNY, meaning the government can justify per-additional-birth subsidies of approximately 139,000–149,000 yuan while remaining fiscally neutral in present value terms. Equivalently, the annuity fiscal value is 4.8–5.2 10k CNY/yr, implying that an annual child-care subsidy of approximately 48,000–52,000 yuan per child for the first three years of life would be fiscally self-financing under the τ^w-adjustment closure. If the policy cannot target only additional births and must be paid to every newborn, the corresponding universal break-even drops to 40,000–54,000 yuan lump-sum (or 14,000–19,000 yuan/yr for three years). The marginal value is the relevant ceiling for a perfectly-targeted incentive; the universal value is the floor when targeting is infeasible.

These results have a clear policy implication: fertility subsidies need not achieve replacement-level fertility to be fiscally justified, provided that the cost of inducing each additional birth remains below the fiscal value. A realistic program that raises TFR from 1.0 to 1.3 generates nearly the same per-birth fiscal return as one that achieves TFR = 2.0, though the total fiscal dividend is naturally smaller. The fiscally justified subsidy level should therefore be calibrated to local fertility responsiveness and to whether the subsidy can target induced births, rather than pegged mechanically to a replacement-level benchmark.

6 Conclusion and Policy Implications

This paper quantifies the fiscal adjustment China's pension system faces under rapid aging and ranks three reforms—delayed retirement, reduced pooling transfers, and higher fertility—by their short- and long-run effects. We build a heterogeneous-agent OLG model of the combined-accounts pension system, calibrated to 2023 moments and validated against the realized 2024 deficit out of sample. Under current policy, the pension deficit widens to 6.4% of GDP in the long run, requiring the wage tax to rise from 9.6% to 27.5%.

The three reform instruments form a clear ranking: reduced pooling delivers substantial short-run relief but cannot restore long-run sustainability and falls disproportionately on low earners; delayed retirement substantially expands the contributor base in both the transition and the long run; higher fertility most directly addresses the demographic root cause—the dependency ratio—and yields the dominant long-run dividend after a two-decade lag. Delayed retirement and higher fertility are complementary because they act on disjoint margins of the pension system: combining delayed retirement to age 65 with replacement-level fertility reduces the terminal wage tax by 11.7 percentage points.

To complement the reform analysis, we compute the *fiscal value of birth*—the present-value fiscal gain per additional birth—as a break-even benchmark for any pro-natalist subsidy. The marginal fiscal value is highest at low fertility near the current TFR of 1.0, so even modest fertility improvements can be fiscally justified if the cost of inducing each additional birth remains below its fiscal value. In the policy-relevant range of TFR = 1.3–1.5, this marginal value provides a fiscal break-even benchmark of approximately 139,000–149,000 yuan per induced birth, equivalent to annual childcare transfers of 48,000–52,000 yuan over the first three years of life.

Two policy implications follow. First, pension reform requires instruments operating at different horizons: delayed retirement and benefit adjustment can relieve near-term fiscal pressure, while higher fertility is needed to improve the long-run demographic base. Second, pro-natalist subsidies should be benchmarked against the fiscal value of induced births and local fertility responsiveness, rather than evaluated solely by whether they restore replacement fertility. This criterion is directly relevant for China's newly implemented national child-rearing subsidy and related local programs.

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Online Appendix

Delayed Retirement or More Births? Short-Run Relief and Long-Run Sustainability of China's Pension System

Not for publication — available as supplementary material

A.1 Solution Method and EGM Derivation

The household problem is solved using the Endogenous Grid Method (EGM), which avoids costly grid search over consumption by exploiting the Euler equation's analytical structure. For each discrete state (b, e, γ) at age j and each candidate savings level a' on the exogenous asset grid, the algorithm: (i) computes next-period states (b', e') from the pension and wage-index transition equations; (ii) interpolates the expected marginal value $\mathbb{E}[\partial V_{j+1}/\partial a']$ from the stored continuation value arrays; (iii) solves the Euler equation for consumption via the inverse marginal utility; and (iv) backs out the *endogenous* current asset level from the budget constraint. The resulting pairs $(a^{\text{endo}}, c^{\text{EGM}})$ are then interpolated onto the exogenous asset grid, with a borrowing-constraint correction applied where the endogenous grid falls below the minimum asset level. The marginal value $\partial V_j/\partial a$ is stored via the envelope condition for the next backward step.

Euler equation in marginal-value form. The code iterates on the marginal value function $\partial V_j/\partial a$ rather than the value function itself. Define $R_t \equiv 1 + r_t(1 - \tau_a)$ as the after-tax gross return on assets and $y_{j,t}$ as non-asset income (wages net of taxes for workers, pension benefits for retirees, plus bequest transfers for eligible ages). For each savings grid point a'_i and discrete state (b, e, γ), the algorithm first computes the expected marginal value

$$E_i = \sum_{\gamma'} \Pi(\gamma'|\gamma) \cdot \frac{\partial V_{j+1}}{\partial a}(a'_i,\, b',\, e',\, \gamma')\,,$$

where b' and e' follow from the pension and wage-index transition equations and do not depend on the consumption–savings choice. The Euler equation in marginal-value form is then

$$c_{j,t}^{-\sigma} = (1+\tau_c) \cdot \beta \big[\alpha_t(j)\, E_i + \big(1-\alpha_t(j)\big)\, \psi_1\, (\psi_2 + a_i')^{-\sigma} \big]\,, \tag{FOC-MV}$$

where $\alpha_t(j)$ is the survival probability and the second term captures the warm-glow bequest motive.

EGM inversion step. Inverting the marginal utility gives EGM consumption $c_i^{\text{egm}} = \text{RHS}_i^{-1/\sigma}$, where RHS_i denotes the right-hand side of (FOC-MV). The *endogenous* current asset level is then recovered from the budget constraint:

$$a_i^{\text{endo}} = \frac{(1+\tau_c)\, c_i^{\text{egm}} + a_i' - y_j}{R_t}. \tag{EGM inversion}$$

The resulting pairs $(a_i^{\text{endo}}, c_i^{\text{egm}})$ are interpolated onto the exogenous asset grid $\{a_k\}$ to obtain the policy function $c^{\text{policy}}(a_k)$.

Borrowing constraint handling. Let $a^{\text{threshold}} = a_0^{\text{endo}}$ be the endogenous asset level corresponding to the smallest savings grid point a'_{min}. For grid points $a_k < a^{\text{threshold}}$, the borrowing constraint $a' \geq a_{\text{min}} = 0$ binds, and consumption is set to

$$c^{\text{constrained}}(a_k) = \frac{R_t\, a_k + y_j - a_{\text{min}}}{1+\tau_c}. \tag{Constrained c}$$

Envelope condition for stored $\partial V/\partial a$. The marginal value passed to the next (younger) age is computed via the envelope theorem:

$$\frac{\partial V_j}{\partial a}(a_k) = \frac{R_t}{1+\tau_c}\, \big(c^{\text{policy}}(a_k)\big)^{-\sigma}. \tag{Stored $\partial V/\partial a$}$$

Since the budget is linear in a and non-asset income y_j does not depend on assets, the envelope condition has a clean closed-form expression, and no automatic differentiation is required.

Interpolation. The urban household state is four-dimensional (a, b, e, γ), requiring trilinear interpolation over (a, b, e) conditional on the discrete shock γ. The rural state is three-dimensional (a, e, γ) with $b = 0$ implicit, so bilinear interpolation over (a, e) suffices.

A.2 Simulation Method and Population Aggregation

The forward side of the solver takes the EGM policy functions from Section A.1 and the exogenous population sequences $\{N_t(j)\}$ as inputs, and produces the cross-sectional distribution of individual states (a, b, e, γ) by age from which the macro aggregates A_t, L_t, C_t, Beq_t, S_t, and B_t are computed. The procedure has three components: (i) a deterministic population projection, (ii) a Monte Carlo household simulation with common random numbers, and (iii) a population-weighted aggregation step that bridges the N_{cohort} simulated agents per age back to macroeconomic aggregates.

Population projection. Population sequences $N_t = (N_t(0), \dots, N_t(J-1))$ are projected forward from the 2023 base distribution by iterating the fertility and survival rules of equations (1) and (2):

$$N_{t+1}(j) = \begin{cases} \sum_{k \in F} \psi_k \, n_{t+1}(k) \, N_{t+1}(k) & j = 0, \\ \alpha_t(j-1) \, N_t(j-1) & 1 \le j \le J-1. \end{cases} \tag{Leslie recursion}$$

Two scenarios are computed with this recursion: a baseline with the 2023 age-specific fertility rates (TFR $\approx$ 1.0), and a counterfactual in which all age-specific rates are rescaled by a common factor so that the total fertility rate rises to replacement level (TFR = 2.0). Projections run for 51 years (2023–2073), after which the age distribution is frozen at the year-50 vector and held constant for the remainder of the model horizon. This freeze ensures that the terminal steady state has a well-defined stationary demographic composition to which the economy can converge.

Common random numbers. The only stochastic element in the household problem is the idiosyncratic productivity shock γ, which follows a discretized three-state Markov chain with transition matrix Π_γ. To eliminate Monte Carlo noise in cross-experiment welfare comparisons, a single set of uniform draws is generated once from a fixed seed and reused across all steady-state and transition simulations. Two tables are pre-drawn: $\varepsilon^{\text{init}} \in [0,1)^{N_{\text{cohort}}}$ for newborn γ initialization, and $\varepsilon^{\gamma} \in [0,1)^{J \times N_{\text{cohort}}}$ for age- and agent-indexed transitions. Given the current productivity $\gamma_{i,j,t}$ and draw $u_{i,j} = \varepsilon^{\gamma}_{j,i}$, the next-period productivity is obtained by inverting the row-CDF of Π_γ:

$$\gamma_{i,j+1,t+1} \;=\; \min\left\{ k \;:\; \sum_{\ell=1}^{k} \Pi_\gamma\big(\gamma_{i,j,t},\, \gamma_\ell\big) \;>\; u_{i,j} \right\}. \tag{CRN}$$

Because the same tables are consumed by every counterfactual, two experiments that produce identical policy functions yield identical simulated trajectories, and welfare differences across experiments are free of Monte Carlo sampling variance.

Cohort simulation for the steady state. For each worker type (urban and rural) the steady-state simulator tracks $N_{\text{cohort}} = 10{,}000$ agents from age 0 to $J-1 = 79$ via a single `jax.lax.scan` over ages. At age 0 each agent is initialized at the borrowing limit $a = a_{\min}$ with empty pension accounts ($b = 0$, $e = 0$) and an initial productivity drawn from $\varepsilon^{\text{init}}$ via the CDF-inversion rule. At each subsequent age j, consumption $c_{i,j}$ is interpolated from the EGM policy $c_j(a, b, e, \gamma)$ (trilinear in (a, b, e) for urban, bilinear in (a, e) for rural), assets are advanced via the budget constraints (5) and (6), the pension account b and the wage index e evolve deterministically via equations (10) and (12), and γ advances via rule (CRN). The scan output is a flat array of $N = J \cdot N_{\text{cohort}} = 800{,}000$ (a, b, e, γ) tuples together with their ages and realized consumption and savings—the steady-state cross-section under the prices and policies passed in.

Single-period evolution for the transition. Along the transition path, the same flat population of N agents is advanced *one period at a time* rather than age by age. Every agent's age increments from j to $j+1$ in parallel; assets and pension-account states are updated using the same budget and transition rules as in the steady state but evaluated at the current period's prices and policies; agents reaching age $j = J$ are replaced in place by newborns with $a = a_{\min}$, $b = e = 0$, and a fresh γ drawn from $\varepsilon^{\text{init}}$. Common-random-number consistency is maintained by indexing the shock tables through $(j_i,\ i \bmod N_{\text{cohort}})$, so each agent's stochastic history is a deterministic function of its identity. The transition scan wraps this single-period update in a second `jax.lax.scan` over the T transition periods, using the policy functions produced by the corresponding backward EGM pass.

Aggregation. Individual decisions are mapped to economy-wide aggregates via population weights. Each simulated agent represents a fraction of the period-t population of its age cohort:

$$w_{i,t} \; = \; \frac{N_t(j_i)}{N_{\text{cohort}}}. \qquad \text{(Population weight)}$$

Within each worker type, the paper's aggregates in equations (18)–(23) are computed as population-weighted sums of the corresponding individual quantities:

$$A_t^s = \sum_i w_{i,t}\, a_{i,t}, \qquad C_t^s = \sum_i w_{i,t}\, c_{i,t}, \qquad L_t^s = \sum_i w_{i,t}\, \ell(j_i)\, \gamma_{i,t}\, \mathbf{1}\{j_i \le j_R\},$$

$$Beq_t^s = \sum_i w_{i,t}\, a'_{i,t}\, \big(1 - \alpha_t(j_i)\big),$$

for $s \in \{u, r\}$. The economy-wide aggregates are obtained by combining the two types with the population shares ϕ_u and $1 - \phi_u$: the aggregator passes $w_{i,t}^u = \phi_u\, N_t(j_i)/N_{\text{cohort}}$ to the urban simulator and $w_{i,t}^r = (1 - \phi_u)\, N_t(j_i)/N_{\text{cohort}}$ to the rural simulator, and sums the two aggregates. Social security expenditure S_t and contributions B_t (equations (21) and (22)) are computed from the same flat arrays using the pension and contribution formulas for each worker type. The urban effective labor L_t^u returned alongside the main aggregates feeds the social average wage $\bar{W}_t = w_t L_t^u / N_t^{\text{workers},u}$ defined in equation (11), which in turn enters the backward EGM pass through the wage-index evolution, closing the loop between the forward and backward halves of the solver.

A.3 Transition Path Algorithms

The transition path algorithm iterates between a backward EGM sweep (solving household problems at each period given price sequences) and a forward Monte Carlo sweep (simulating 800,000 agents through the transition). The outer loop updates price sequences via damped iteration until convergence. Both sweeps are implemented as single `jax.lax.scan` calls for computational efficiency. We describe the two fiscal closure variants below.

In both algorithms, the damping parameter s follows a dynamic schedule that starts near 1 (conservative updates) and decreases toward 0.3 as the iteration progresses, which improves stability in the early iterations while allowing faster convergence later. The bisection bracket in Algorithm 1 is warm-started from the previous price iteration, narrowing the search range and typically requiring fewer than 10 bisection steps per price iteration. Convergence of the outer price loop is typically achieved within 30–50 iterations.

A.4 Calibration Validation

The discretizations and exogenous schedules underpinning the calibration are summarized in Figures A.1–A.3. They document, respectively, the age-specific fertility rates fed into the UN-based population projection, the three-state discretization of the productivity shock γ, and the age-survival probabilities $\alpha_t(j)$ underlying the lifecycle structure.

Algorithm 1 τ^w-Adjustment Transition Path

Require: Initial SS, terminal SS, demographic sequence $\{N_t\}$, target D_T/Y_T
Ensure: Equilibrium price and policy sequences $\{r_t, w_t, \tau_t^w\}$

Initialize $\{r_t, w_t\}$ by linear interpolation between initial and terminal SS
Initialize $\{beq_t, \bar{W}_t\}$ by linear interpolation
Set bisection bracket $[\tau_{\text{lo}}^w, \tau_{\text{hi}}^w]$
for $n = 1, 2, \dots$ (price iteration) **do**
  for $m = 1, 2, \dots$ (bisection on τ_{trans}^w) **do**
    $\tau_{\text{trans}}^w \leftarrow (\tau_{\text{lo}}^w + \tau_{\text{hi}}^w)/2$
    Set $\tau_t^w = \tau_{\text{trans}}^w$ for $t < T_{\text{trans}}$; $\tau_t^w = \tau_{\text{term}}^w$ for $t \geq T_{\text{trans}}$
    Backward EGM sweep (`jax.lax.scan`): solve household problems from $t = T-1$ to $t = 0$ using stored terminal-SS $\partial V/\partial a$ as boundary condition
    Forward MC sweep (`jax.lax.scan`): simulate agents from $t = 0$ to $t = T-1$ starting from initial-SS distribution
    Aggregate: $\{A_t, L_t, C_t, Beq_t, S_t, B_t\}$
    Compute debt dynamics: $D_0 = D^{\text{init}}$; for each t, compute tax revenue $T_t = \tau_t^w w_t L_t + \tau_a r_t A_t + \tau_c C_t$, government spending $G_t = (G/Y) \cdot Y_t$, and update $D_{t+1} = (1+r_t)D_t + G_t + S_t - B_t - T_t$
    if $|D_T/Y_T - \text{target}| < \varepsilon_D$ **then**
      break (bisection converged)
    else if $D_T/Y_T > \text{target}$ **then**
      $\tau_{\text{lo}}^w \leftarrow \tau_{\text{trans}}^w$
    else
      $\tau_{\text{hi}}^w \leftarrow \tau_{\text{trans}}^w$
    end if
  end for
  Compute implied prices: $r_t^{\text{new}} = \alpha Z K_t^{\alpha-1} L_t^{1-\alpha} - \delta$, $w_t^{\text{new}} = (1-\alpha) Z K_t^{\alpha} L_t^{-\alpha}$, where $K_t = A_t - D_t$
  Damped update: $r_t \leftarrow s \cdot r_t + (1-s) \cdot r_t^{\text{new}}$, $w_t \leftarrow s \cdot w_t + (1-s) \cdot w_t^{\text{new}}$
  if $\frac{1}{T} \sum_t \left[(r_t - r_t^{\text{new}})^2 + (w_t - w_t^{\text{new}})^2 \right] < 10^{-5}$ **then**
    return converged solution
  end if
end for

Algorithm 2 G-Adjustment Transition Path

Require: Initial SS, terminal SS, demographic sequence $\{N_t\}$, fixed D/Y ratio
Ensure: Equilibrium price sequence $\{r_t, w_t\}$ and government spending $\{G_t\}$

Initialize $\{r_t, w_t\}$ by linear interpolation between initial and terminal SS
Initialize $\{beq_t, \bar{W}_t\}$ by linear interpolation
Set $\tau_t^w = \tau_0^w$ (fixed at initial value) for all t
for $n = 1, 2, \ldots$ (price iteration) **do**
    Backward EGM sweep (`jax.lax.scan`): solve household problems from $t = T-1$ to $t = 0$
    Forward MC sweep (`jax.lax.scan`): simulate agents from $t = 0$ to $t = T-1$
    Aggregate: $\{A_t, L_t, C_t, Beq_t, S_t, B_t\}$
    For each t: compute $K_t = A_t - D_t$ where $D_t = (D/Y) \cdot Y_t$, tax revenue $T_t = \tau_0^w w_t L_t + \tau_a r_t A_t + \tau_c C_t$, and residual government spending $G_t = T_t + B_t - (1+r_t)D_t - S_t + D_{t+1}$
    Compute implied prices: $r_t^{\text{new}} = \alpha Z K_t^{\alpha-1} L_t^{1-\alpha} - \delta, \quad w_t^{\text{new}} = (1-\alpha) Z K_t^{\alpha} L_t^{-\alpha}$
    Damped update: $r_t \leftarrow s \cdot r_t + (1-s) \cdot r_t^{\text{new}}, \quad w_t \leftarrow s \cdot w_t + (1-s) \cdot w_t^{\text{new}}$
    if $\frac{1}{T}\sum_t \left[(r_t - r_t^{\text{new}})^2 + (w_t - w_t^{\text{new}})^2\right] < 10^{-5}$ **then**
        return converged solution
    end if
end for

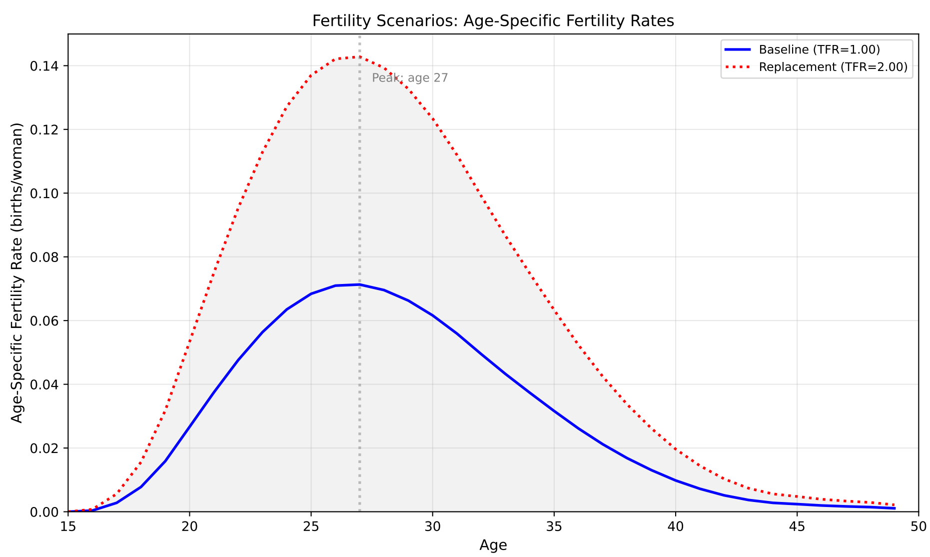

Figure A.1: Age-specific fertility rates underlying the population projection. Solid line: 2023 baseline; dashed: replacement-level (TFR $= 2.0$) counterfactual.

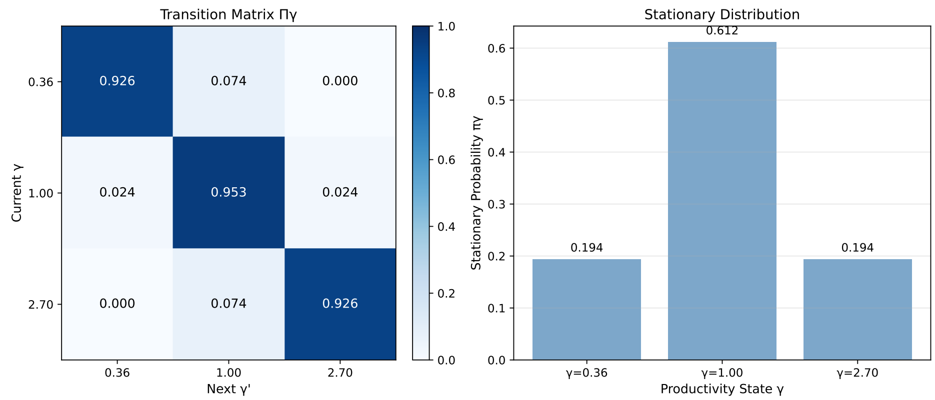

Figure A.2: Discretized productivity shock $\gamma \in \{0.36, 1.0, 2.70\}$ and its three-state Markov transition matrix.

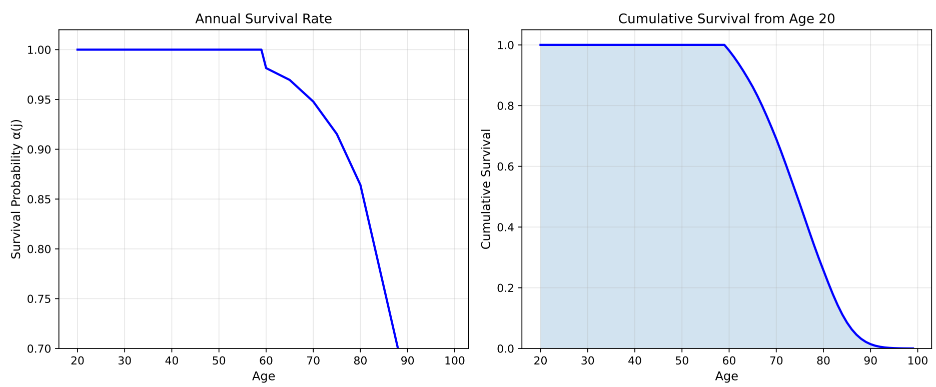

Figure A.3: Age-conditional survival probability $\alpha(j)$ used in the household lifecycle problem.

A.5 G-Adjustment Sensitivity

The main paper analyses the demographic transition under the τ^w-adjustment closure, in which the wage tax adjusts to anchor the debt-to-GDP target. This subsection reports the corresponding results under an alternative closure in which all tax rates are held at their 2023 values and government spending G_t adjusts residually (equation (26)). The G-adjustment closure is computationally convenient and avoids the tax-distortion channel; it is reported as a less-distortionary fiscal-policy reference, not as an alternative main analysis.

Terminal steady state under G-adjustment. Holding the wage tax at its initial 9.60% value, government purchases must fall from 21.80% to 12.92% of GDP at the terminal steady state to maintain the debt-to-GDP cap—a roughly 41% reduction in the public-goods share. The capital–output ratio rises to 7.82 (versus 6.94 under τ^w-adjustment), since the absence of the tax wedge preserves household incentives for saving.

Table A.1: Terminal Steady-State Aggregates: G-Adjustment

Variable	Value	Variable	Value
Wage tax τ^w	9.60%	Capital–output ratio K/Y	7.821
Interest rate r	0.0239	Debt–output ratio D/Y	23.80%
Wage w	1.1844	Gov. purchases G/Y	12.92%

Table A.2: Fiscal Burden Decomposition under G-Adjustment

	G-Adjustment
Initial (2023)	$G/Y = 21.80\%$
Counterfactual (2023 demog., balanced)	$G/Y = 18.76\%$
Terminal (aged demog.)	$G/Y = 12.92\%$
Debt burden	−3.04pp
Aging burden	−5.84pp
Total adjustment	−8.88pp

Transition path under G-adjustment. With the wage tax fixed, government purchases share declines monotonically toward the terminal 12.92% value, with the minimum during the demographic-shock window ($\approx$ 12.95%) essentially equal to the terminal level. The debt-to-GDP ratio remains anchored at 23.80% throughout.

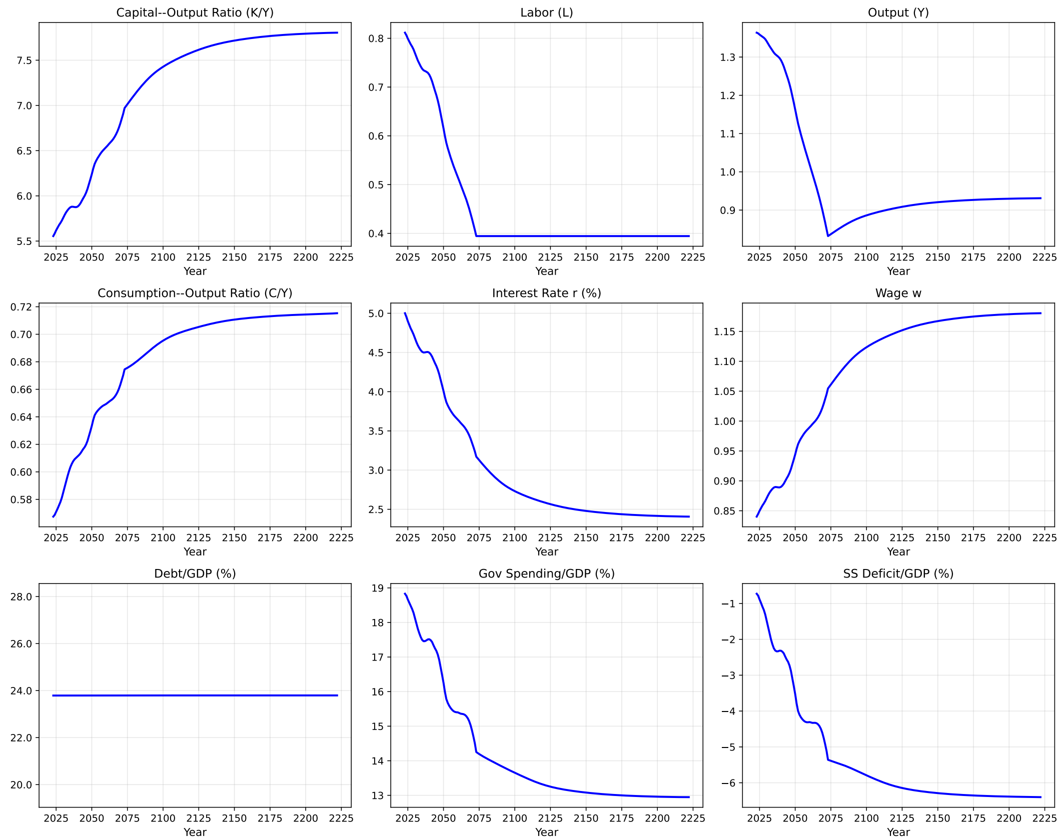

Figure A.4: Transition paths under G-adjustment.

Policy experiments under G-adjustment. Table A.3 and Figure A.5 report the four policy experiments under the G-adjustment sensitivity. The ranking mirrors the τ^w-adjustment findings; only the instrument differs. Replacement-level fertility raises terminal G/Y the most among single instruments (+4.6 pp to 17.51%); delayed retirement to age 65 contributes +2.7 pp (+3.1 pp at age 70); reduced pooling +1.9 pp. Combining delayed retirement to age 65 with replacement-level fertility raises terminal G/Y by 5.9 pp to 18.86%, nearly restoring the initial level of public spending.

Table A.3: Policy Experiment Summary: G-Adjustment

Experiment	G/Y^{term}	$\overline{G/Y}$	$\min(G/Y)$	$\Delta G/Y^{term}$
Baseline	12.92%	14.11%	12.95%	—
Delayed Retirement (+5 years)	15.59%	16.46%	15.64%	+2.7pp
Reduced Pooling ($\alpha_b = 0.5\%$)	14.78%	15.85%	14.83%	+1.9pp
Replacement TFR (2.0)	17.51%	17.47%	16.45%	+4.6pp
Combined (Delayed Ret. + Repl. TFR)	18.86%	18.79%	17.59%	+5.9pp

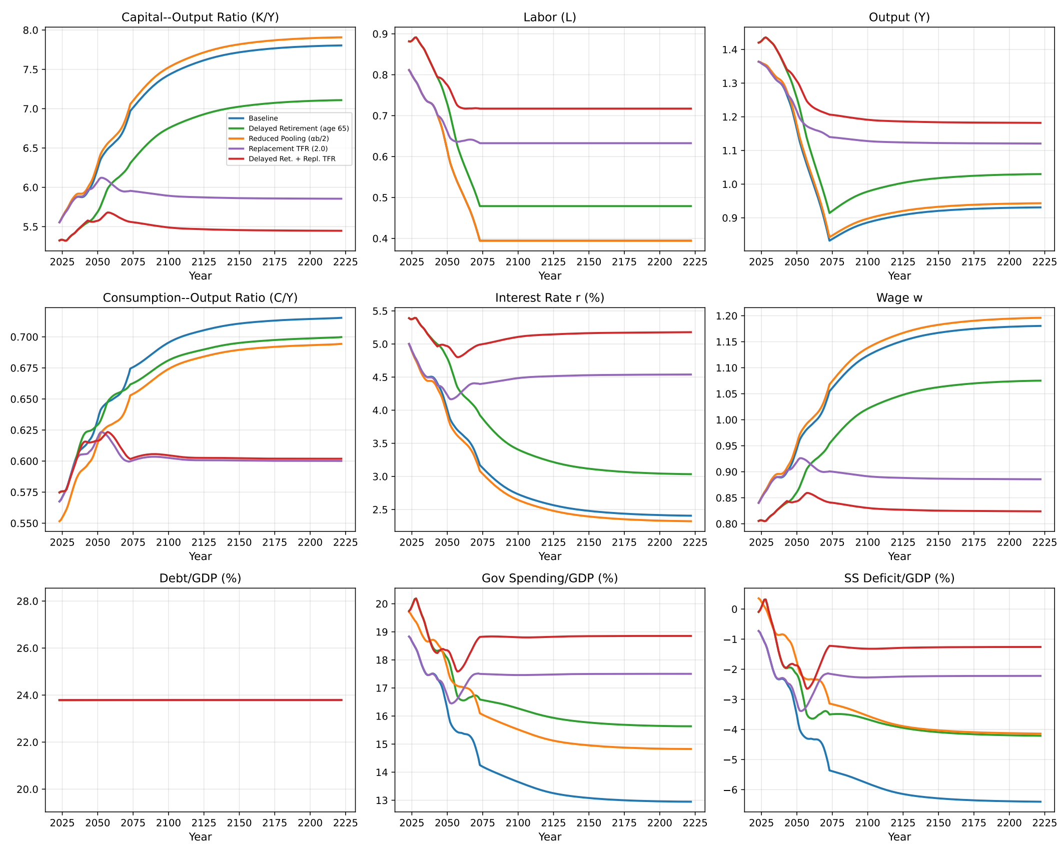

Figure A.5: Policy experiment comparison: G-adjustment.

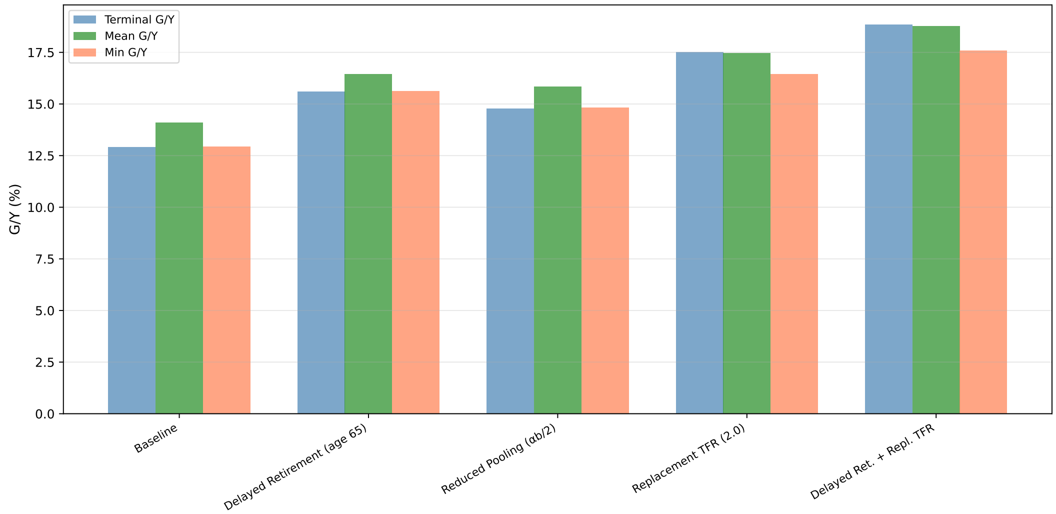

Figure A.6: Fiscal decomposition under G-adjustment.

Fiscal value of birth under G-adjustment. Under G-adjustment, the lump-sum fiscal value of an additional birth at replacement TFR is 21.8 10k CNY (7.6 10k CNY/yr annuity over 3 years), and at TFR = 1.3–1.5 the marginal value is 24.4–25.4 10k CNY (8.4–8.8 10k CNY/yr). These values are systematically larger than under τ^w-adjustment because the G-adjustment closure does not impose the tax-distortion drag on the saving channel that compresses the fiscal dividend under τ^w.

Welfare and intergenerational incidence under G-adjustment. Under G-adjustment, the urban–rural welfare differential is substantially larger than under τ^w-adjustment (urban +68.35% versus rural +58.29% in CEV terms, Table A.4). We caution that absolute CEV levels under this closure are sensitive to the omission of any direct utility value for government consumption G in the household problem: the collapse of G/Y from 21.8% to 12.9% shows up in the model as a welfare gain because households retain more income, but the foregone public goods would offset this to an unknown degree. The cross-type and cohort-level *differentials* are robust to this caveat because urban, rural, and cohort agents face the same G path within a closure. Figure A.7 shows the cohort-level incidence of each reform under G-adjustment.

Table A.4: Welfare CEV under G-Adjustment

	G-adjustment
Overall CEV	+60.40%
Urban	+68.35%
Rural	+58.29%

Table A.5: Fiscal Value of Birth under G-Adjustment

	TFR vs Baseline	TFR + Ret. vs Ret.
Lump-sum FV (10k CNY)	21.8	15.6
Annuity FV (10k CNY/yr, 3 yr)	7.6	5.4

Table A.6: TFR Sweep Fiscal Values under G-Adjustment (10k CNY)

	Lump-sum FV		Annuity FV (3 yr)	
TFR	Marginal	Universal	Marginal	Universal
1.1	27.8	3.0	9.63	1.03
1.3	25.4	6.8	8.82	2.38
1.5	24.4	9.5	8.44	3.28
1.8	22.6	11.7	7.83	4.05
2.0	21.5	12.5	7.45	4.33

Table A.7: Inequality under G-Adjustment

	Initial (2023)	Terminal (G-adj)
Gini coefficients		
Wealth	0.459	0.407
Urban	0.453	0.414
Rural	0.440	0.362
Consumption	0.309	0.263
Income	0.388	0.427
Wealth distribution		
P90/P10	16.3	10.1
Top 10% share	30.9%	27.3%
Bottom 50% share	18.5%	21.6%

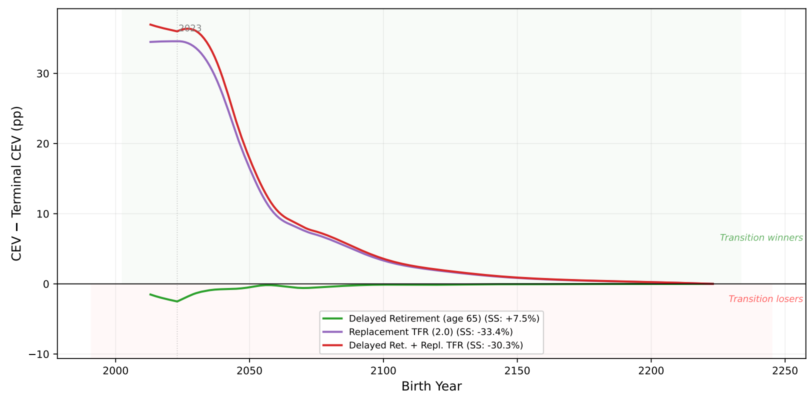

Figure A.7: Cohort CEV by birth year for each reform relative to the baseline demographic transition under G-adjustment, in percentage points and de-meaned by the terminal (last-cohort) CEV. The intergenerational incidence is markedly smoother than under τ^w-adjustment because the fiscal adjustment falls on public spending rather than on labor income, muting within-cohort redistribution through the tax wedge.

A.6 Supplementary Results

Delayed-retirement effects under the statutory formula. The statutory annuity divisor $M(j_R)/12$ in Table A.8 is shorter than the model-implied expected post-retirement lifespan

$$E_{j_R} \equiv \sum_{k=0}^{J-1-j_R} \prod_{m=0}^{k-1} \alpha(j_R + m),$$

under the calibrated survival schedule $\alpha(\cdot)$. Figure A.8 plots the implied *personal-account claim ratio*

$$\rho(j_R) = \frac{E_{j_R}}{M(j_R)/12},$$

the expected lifetime payout per unit of accumulated balance b. The undiscounted ratio rises from $\rho(60) \approx 1.37$ to $\rho(70) \approx 2.03$ — i.e. for every yuan saved into the personal account, the statutory formula pays out 1.37 yuan in expectation under retirement at age 60, and 2.03 yuan under retirement at age 70. This positive-and-rising profile reflects the fact that $M(j_R)/12$ shrinks faster than E_{j_R} does as j_R rises: the State Council annuity divisor is calibrated to a fixed life-table assumption that gives less weight to long-tail survival than our calibrated $\alpha(\cdot)$. Discounting at the calibrated notional account return $r_{ss} = 2.91\%$

Table A.8: Statutory Personal-Account Annuity Months $M(j_R)$ (State Council Document [2005] No. 38)

Retirement age	Months	Retirement age	Months
40	233	56	164
41	230	57	158
42	226	58	152
43	223	59	145
44	220	60	139
45	216	61	132
46	212	62	125
47	207	63	117
48	204	64	109
49	199	65	101
50	195	66	93
51	190	67	84
52	185	68	75
53	180	69	65
54	175	70	56
55	170		

Note: Retirement age is in real-age units. Months $M(j_R)$ enter the urban pension formula (8) as the annuity divisor $M(j_R)/12$ (years). Source: State Council Document [2005] No. 38, Personal-Account Pension Annuity Months Schedule.

shifts the curve down only mildly and leaves it everywhere above unity for $j_R \geq 65$. Two implications follow. First, the personal-account formula carries an implicit subsidy that is not neutralised — and is in fact *widened* per retiree — by delayed retirement. Second, the aggregate fiscal saving from delayed retirement on the personal-account side comes through the extensive margin (fewer retirees and fewer claim-years) rather than through the intensive margin (per-retiree generosity).

Figure A.9 plots the corresponding expected lifetime pooling claim per unit of historical wage,

$$\Pi(j_R) \;=\; \alpha_b \, j_R \, E_{j_R},$$

which moves with two opposing forces under delayed retirement: the contribution-year multiplier j_R rises while residual longevity E_{j_R} falls. The product is the cleanest summary of the pooling-side mechanical lever; in our calibration $\Pi(j_R)$ declines roughly linearly from 6.33 years-of-e at age 60 to 4.73 years-of-e at age 70 — about a 25% reduction in lifetime pooling liability per retiree. The per-period pooling $\alpha_b j_R$, displayed on the second axis, rises monotonically and explains the per-retiree benefit increase noted in Section 4.5; the lifetime liability nonetheless falls because the residual-longevity decline dominates. Taken together, these two figures decompose the per-retiree mechanical effect of delayed retirement: the personal-account formula becomes *more* actuarially generous (because $M(j_R)/12$ shrinks faster than E_{j_R}), while the pooling formula becomes *less* costly per retiree on a lifetime basis (because E_{j_R} shrinks faster than j_R rises). The aggregate fiscal effect documented in the main text combines these mechanical channels with the demographic extensive margin (population shifts between contributors and retirees).

Aggregate terminal-SS welfare CEV. Table A.12 reports the aggregate consumption-equivalent variation comparing each terminal steady state to the 2023 initial condition. The differential measures emphasized in the main text—urban vs. rural within a closure, and cohort-level differentials across reforms within a closure—are robust to the model's omission

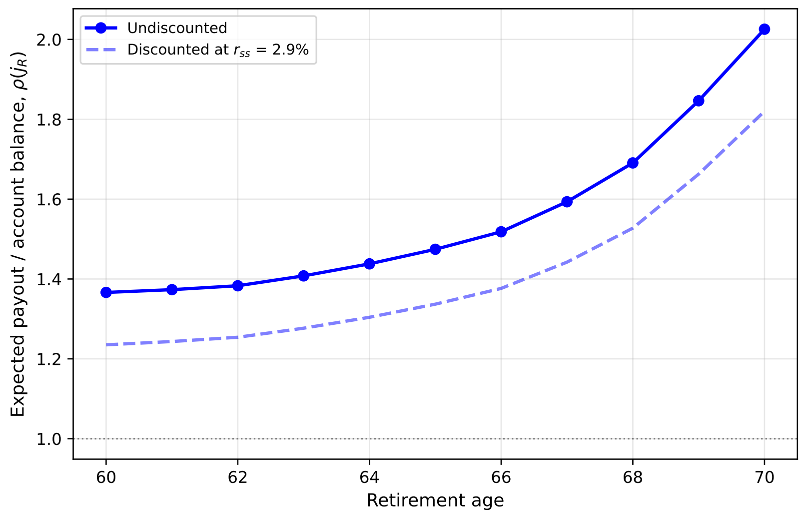

Figure A.8: Personal-account claim ratio $\rho(j_R) = E_{j_R}/(M(j_R)/12)$. Solid: undiscounted expected lifetime payout per unit of accumulated balance b. Dashed: discounted at the notional account return r_{ss}. The horizontal dotted line at 1.0 marks actuarial fairness; values above the line indicate an implicit personal-account subsidy.

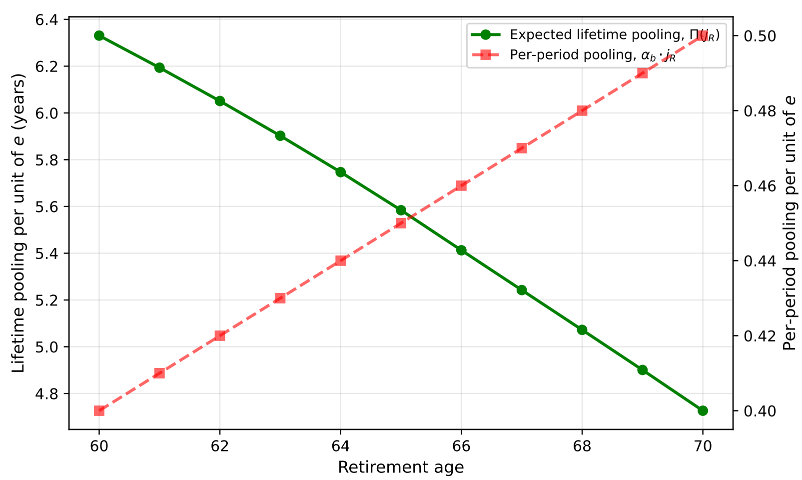

Figure A.9: Pooling claim under the statutory pooling formula. Left axis (green): expected lifetime pooling per unit of historical wage e, $\Pi(j_R) = \alpha_b\, j_R\, E_{j_R}$. Right axis (red): per-period pooling, $\alpha_b\, j_R$. The two axes show the opposing per-period vs. lifetime effects of delayed retirement on the pooling component.

Table A.9: Social Security Deficit Decomposition: Urban vs. Rural

	Initial (2023)	Terminal (τ^w-adj)
Pension expenditure S/Y		
Urban	5.02%	10.43%
Rural	0.27%	0.55%
Total	5.29%	10.98%
Pension revenue B/Y		
Urban	4.47%	4.47%
Rural	0.09%	0.07%
Total	4.56%	4.54%
Pension deficit $(B-S)/Y$		
Urban	−0.55%	−5.96%
Rural	−0.18%	−0.48%
Total	−0.73%	−6.44%

Table A.10: SS Deficit Decomposition over Transition (% of GDP)

Sector	2023	2033	2043	2053	2073
Urban	+0.55%	+1.80%	+2.29%	+3.88%	+5.24%
Rural	+0.19%	+0.25%	+0.27%	+0.35%	+0.42%
Total	+0.74%	+2.05%	+2.56%	+4.24%	+5.65%

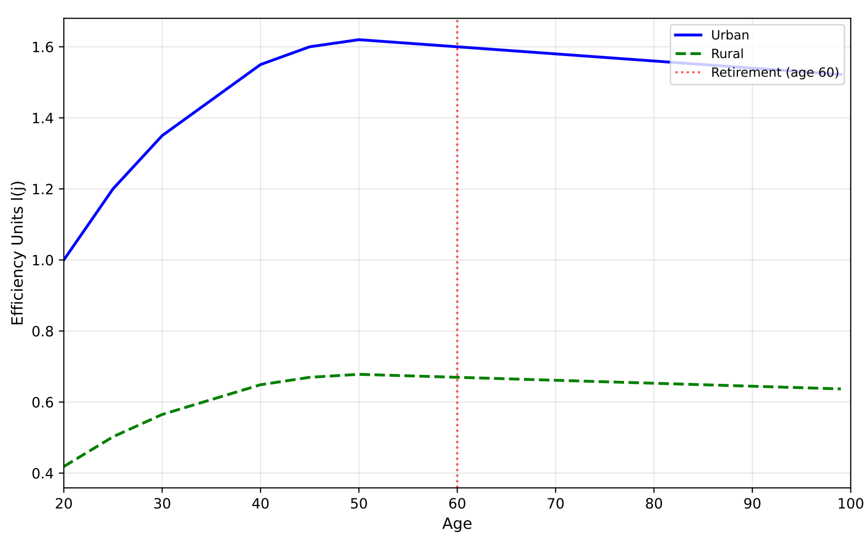

Figure A.10: Age-Efficiency Profile: Urban vs. Rural Workers

Table A.11: Consumption and Wealth by Age Group: Steady-State Comparison

	Initial (2023)	Terminal (τ^w-adj)
Mean consumption		
Young (20–39)	0.732	0.941
Old (40–59)	1.082	1.202
Retired (60+)	1.199	1.212
Std. dev. of consumption		
Young (20–39)	0.343	0.321
Old (40–59)	0.609	0.552
Retired (60+)	0.698	0.612
Gini of consumption		
Young (20–39)	0.246	0.181
Old (40–59)	0.297	0.244
Retired (60+)	0.311	0.273
Mean wealth		
Young (20–39)	4.811	7.236
Old (40–59)	13.982	17.343
Retired (60+)	11.778	11.885
Std. dev. of wealth		
Young (20–39)	4.828	5.739
Old (40–59)	9.590	9.613
Retired (60+)	9.924	9.769
Gini of wealth		
Young (20–39)	0.500	0.423
Old (40–59)	0.347	0.289
Retired (60+)	0.427	0.427

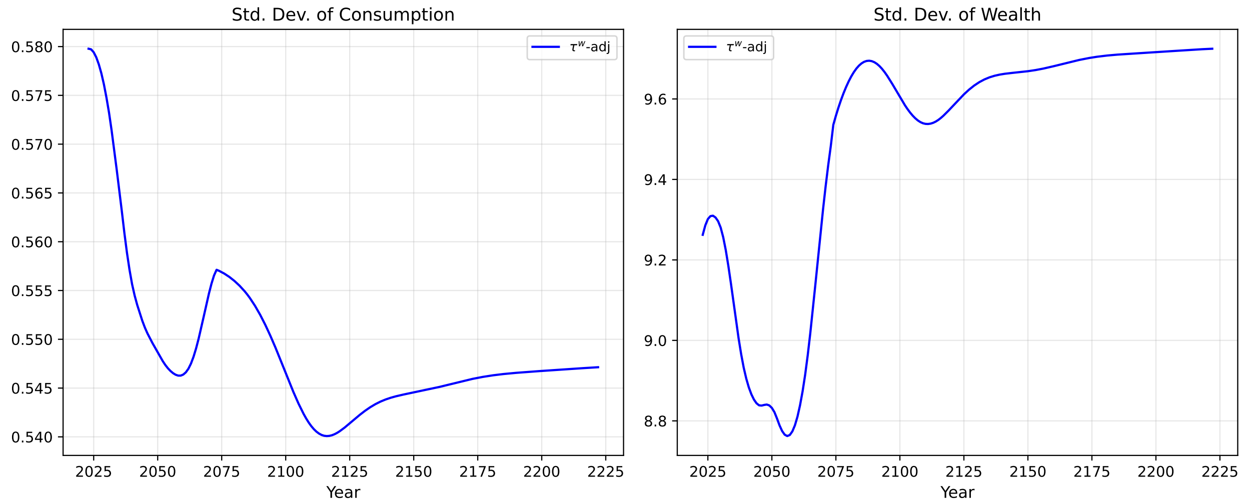

Figure A.11: Standard deviation of consumption (left) and wealth (right) along the transition path under τ^w-adjustment and G-adjustment.

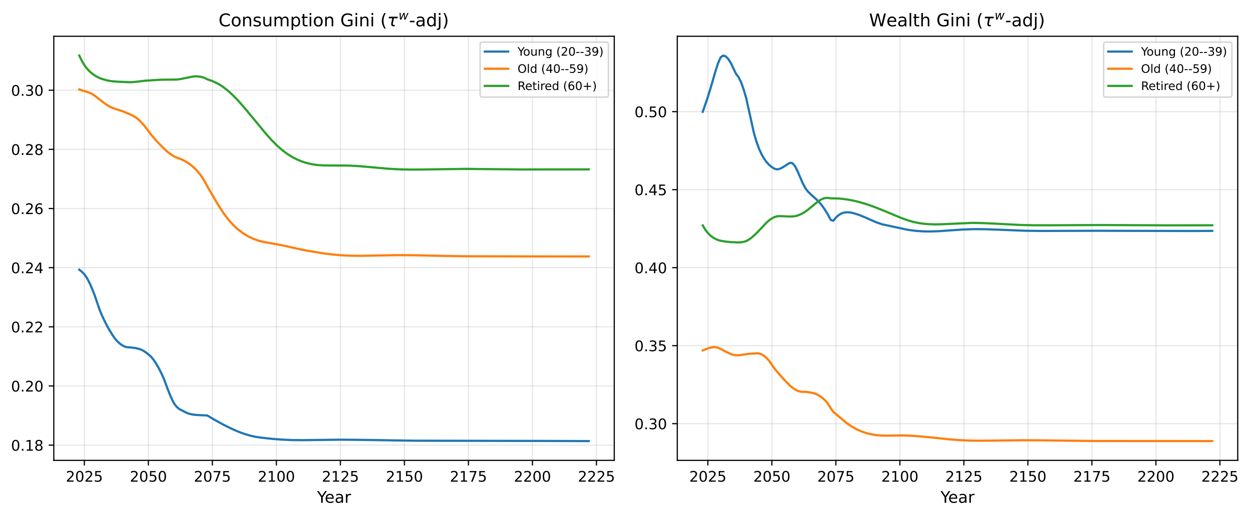

Figure A.12: Gini coefficients of consumption (top) and wealth (bottom) by age group along the transition path. Left: τ^w-adjustment; right: G-adjustment.

of any direct utility value for government consumption G, because the (unmeasured) G contribution cancels in such comparisons. The absolute CEV levels in Table A.12, in contrast, are sensitive to this omission and should be read as upper bounds. The reform-level policy CEV table (with a finer urban/rural/all decomposition) is retained in the internal additional-results document rather than reproduced here.

Table A.12: Welfare: Consumption Equivalent Variation (CEV)

	τ^w-adjustment
Overall CEV	+27.63%
Urban	+26.63%
Rural	+27.92%

Note: CEV measures the permanent proportional change in consumption needed in the initial condition to match lifetime utility in the terminal steady state. Positive values indicate welfare gain in the terminal state. Absolute CEV levels are sensitive to the G-utility omission discussed in Section 4.6; the urban–rural differential within each fiscal mode is robust to that caveat and is the object emphasized in the main text.

Table A.13: Sustainability Comparison: Delayed Retirement to Age 65 vs. Age 70

Metric	Baseline	Ret. age 65	Ret. age 70
τ_w during transition	19.63%	16.35%	15.02%
τ_w at terminal SS	27.50%	22.40%	21.77%
K/Y at terminal SS	6.94	6.63	6.38
$(S-B)/Y$ at terminal SS	6.44%	4.24%	4.28%
$(S-B)/Y$ peak during transition	6.43%	4.23%	4.28%

Note: Each column reports outcomes under one retirement policy applied to the demographic transition. Panel A holds G/Y fixed at the 2023 calibration target and lets the wage tax τ_w adjust to satisfy the long-run debt rule; Panel B holds τ_w fixed and lets G/Y absorb the fiscal residual. $(S-B)/Y$ is the pension-system deficit (positive values indicate benefits exceeding contributions).

Table A.14: Sustainability Across the Fertility Sweep: TFR-Only and TFR + Delayed Retirement (age 65), Both Fiscal Modes

TFR	*TFR only* τ_w^{trans}	*TFR + Delayed Ret. (age 65)* τ_w^{trans}
1.1	19.56%	16.31%
1.2	19.51%	16.30%
1.3	19.42%	16.26%
1.4	19.36%	16.23%
1.5	19.29%	16.17%
1.6	19.22%	16.13%
1.7	19.14%	16.08%
1.8	19.08%	16.03%
1.9	19.01%	15.98%
2.0	18.93%	15.93%

Note: Each row reports outcomes under one target TFR. "TFR only" columns apply the fertility scenario in isolation; "TFR + Delayed Ret." columns combine the fertility scenario with retirement-age delay to 65. τ_w^{trans} is the constant wage tax during the transition phase under τ_w-adjustment. $\Delta(G/Y)_{t=50}$ and $\Delta(G/Y)_{LR}$ are the differences (in percentage points of Y) of G/Y in 2073 and at the terminal steady state, respectively, relative to the corresponding no-fertility-reform reference path under G-adjustment.